Heat and Mass Transfer: Fundamentals & Applications
Fourth Edition
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Chapter 6
FUNDAMENTALS OF CONVECTION

Objectives

- Understand the physical mechanism of convection and its classification
- Visualize the development of velocity and thermal boundary layers during flow over surfaces
- Gain a working knowledge of the dimensionless Reynolds, Prandtl, and Nusselt numbers
- Distinguish between laminar and turbulent flows, and gain an understanding of the mechanisms of momentum and heat transfer in turbulent flow
- Derive the differential equations that govern convection on the basis of mass, momentum, and energy balances, and solve these equations for some simple cases such as laminar flow over a flat plate
- Nondimensionalize the convection equations and obtain the functional forms of friction and heat transfer coefficients
- Use analogies between momentum and heat transfer, and determine heat transfer coefficient from knowledge of friction coefficient

PHYSICAL MECHANISM OF CONVECTION

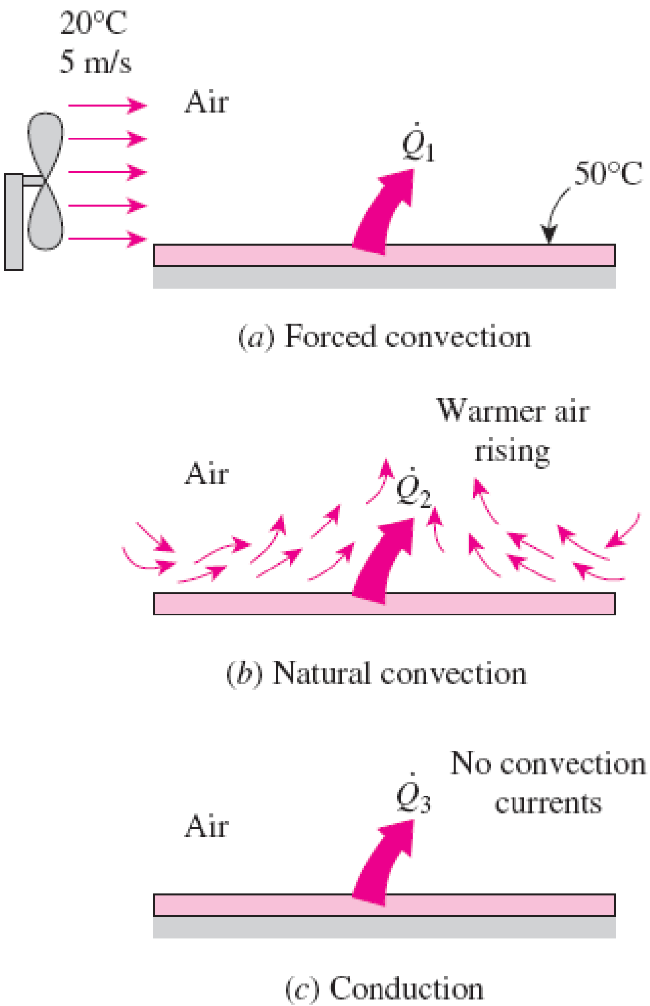


FIGURE 6–1
Heat transfer from a hot surface to the surrounding fluid by convection and conduction.

Conduction and convection both require the presence of a material medium but convection requires fluid motion.

Convection involves fluid motion as well as heat conduction.

Heat transfer through a solid is always by conduction.

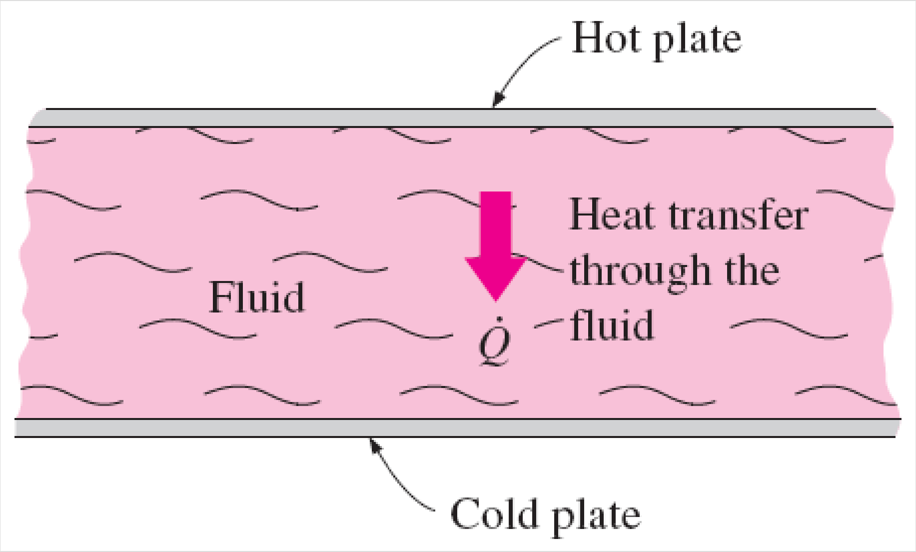
Heat transfer through a fluid is by convection in the presence of bulk fluid motion and by conduction in the absence of it.

Therefore, conduction in a fluid can be viewed as the limiting case of convection, corresponding to the case of quiescent fluid.

The fluid motion enhances heat transfer, since it brings warmer and cooler chunks of fluid into contact, initiating higher rates of conduction at a greater number of sites in a fluid.

The rate of heat transfer through a fluid is much higher by convection than it is by conduction.

In fact, the higher the fluid velocity, the higher the rate of heat transfer.



Heat transfer through a fluid sandwiched between two parallel plates.

Convection heat transfer strongly depends on the fluid properties *dynamic viscosity*, *thermal conductivity, density*, and *specific heat*, as well as the *fluid velocity*. It also depends on the *geometry* and the *roughness* of the solid surface, in addition to the *type of fluid flow* (such as being streamlined or turbulent).

$$\dot{q}_{\text{conv}} = h(T_s - T_\infty) \qquad (\text{W/m}^2)$$

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) \qquad (\text{W})$$

Newton's law of cooling

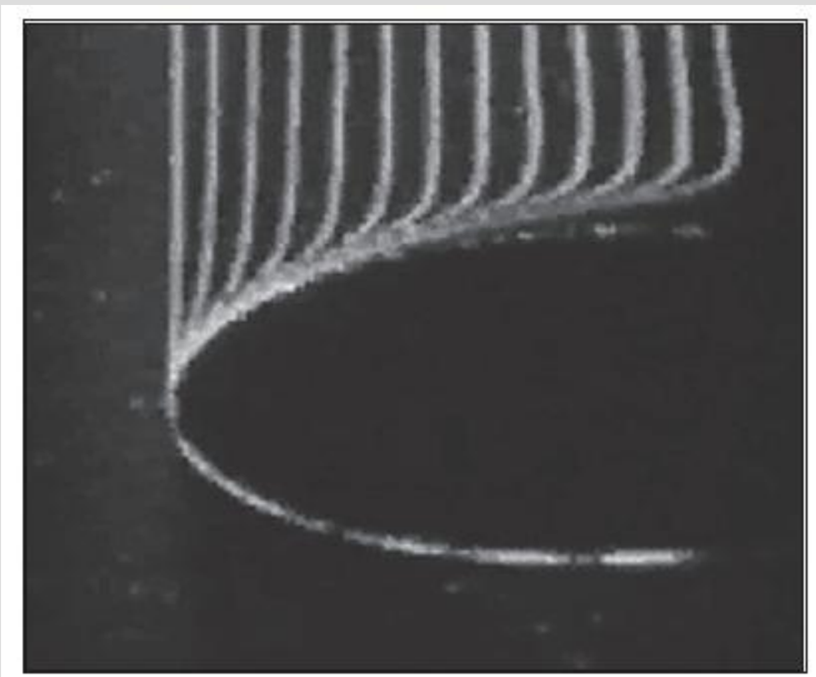
h = convection heat transfer coefficient, $\text{W/m}^2 \cdot {}^\circ\text{C}$
A_s = heat transfer surface area, m^2
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C

Convection heat transfer coefficient, *h*: The rate of heat transfer between a solid surface and a fluid per unit surface area per unit temperature difference.

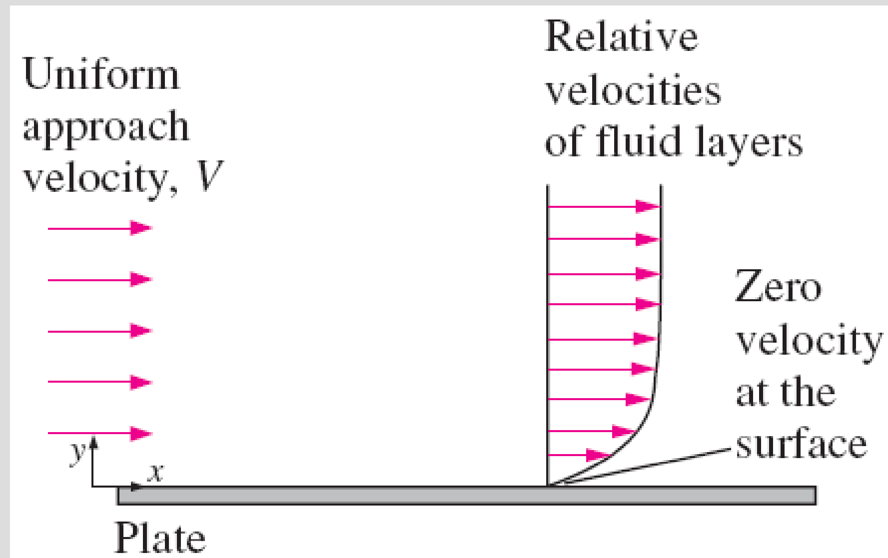
No-slip condition: A fluid in direct contact with a solid “sticks” to the surface due to viscous effects, and there is no slip.

Boundary layer: The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant.

The fluid property responsible for the no-slip condition and the development of the boundary layer is *viscosity.*



The development of a velocity profile due to the no-slip condition as a fluid flows over a blunt nose.



A fluid flowing over a stationary surface comes to a complete stop at the surface because of the no-slip condition.

An implication of the no-slip condition is that heat transfer from the solid surface to the fluid layer adjacent to the surface is by *pure conduction,* since the fluid layer is motionless, and can be expressed as

$$\dot{q}_{\text{conv}} = \dot{q}_{\text{cond}} = -k_{\text{fluid}} \left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad (\text{W/m}^2)$$

The determination of the *convection heat transfer coefficient* when the temperature distribution within the fluid is known

$$h = \frac{-k_{\text{fluid}}(\partial T/\partial y)_{y=0}}{T_s - T_\infty} \qquad (\text{W/m}^2 \cdot {}^\circ\text{C})$$

The convection heat transfer coefficient, in general, varies along the flow (or *x*-) direction. The *average* or *mean* convection heat transfer coefficient for a surface in such cases is determined by properly averaging the *local* convection heat transfer coefficients over the entire surface area A_s or length L as

$$h = \frac{1}{A_s}\int_{A_s} h_{\text{local}} dA_s \qquad \text{and} \qquad h = \frac{1}{L}\int_0^L h_x dx$$

Wilhelm Nusselt (1882–1957), was a German engineer, born in Nuremberg, Germany. He studied machinery at the Technical Universities of Berlin-Charlottenburg and Munchen and conducted advanced studies in mathematics and physics. His doctoral thesis was on the "Conductivity of Insulating Materials" which he completed in 1907. In 1915, Nusselt published his pioneering paper: The Basic Laws of Heat Transfer, in which he first proposed the dimensionless groups now known as the principal parameters in the similarity theory of heat transfer. His other famous works were concerned with the film condensation of steam on vertical surfaces, the combustion of pulverized coal and the analogy between heat and mass transfer in evaporation. Among his well known mathematical works are the solutions for laminar heat transfer in the entrance region of tubes and for heat exchange in cross-flow, and the basic theory of regenerators.

Nusselt Number

In convection studies, it is common practice to nondimensionalize the governing equations and combine the variables, which group together into *dimensionless numbers* in order to reduce the number of total variables.

Nusselt number: *Dimensionless convection heat transfer coefficient*

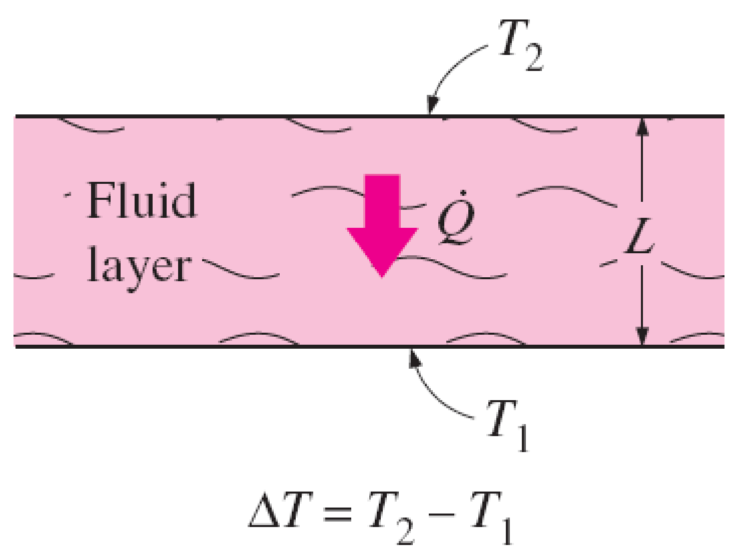
$$\mathrm{Nu} = \frac{hL_c}{k}$$

L_c characteristic length

$$\dot{q}_{\mathrm{conv}} = h\Delta T$$

$$\dot{q}_{\mathrm{cond}} = k\frac{\Delta T}{L}$$

$$\frac{\dot{q}_{\mathrm{conv}}}{\dot{q}_{\mathrm{cond}}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \mathrm{Nu}$$



Heat transfer through a fluid layer of thickness L and temperature difference ΔT.

The Nusselt number represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer.

The larger the Nusselt number, the more effective the convection.

A Nusselt number of Nu = 1 for a fluid layer represents heat transfer across the layer by pure conduction.

Convection in daily life

FIGURE 6–7

We resort to forced convection whenever we need to increase the rate of heat transfer.

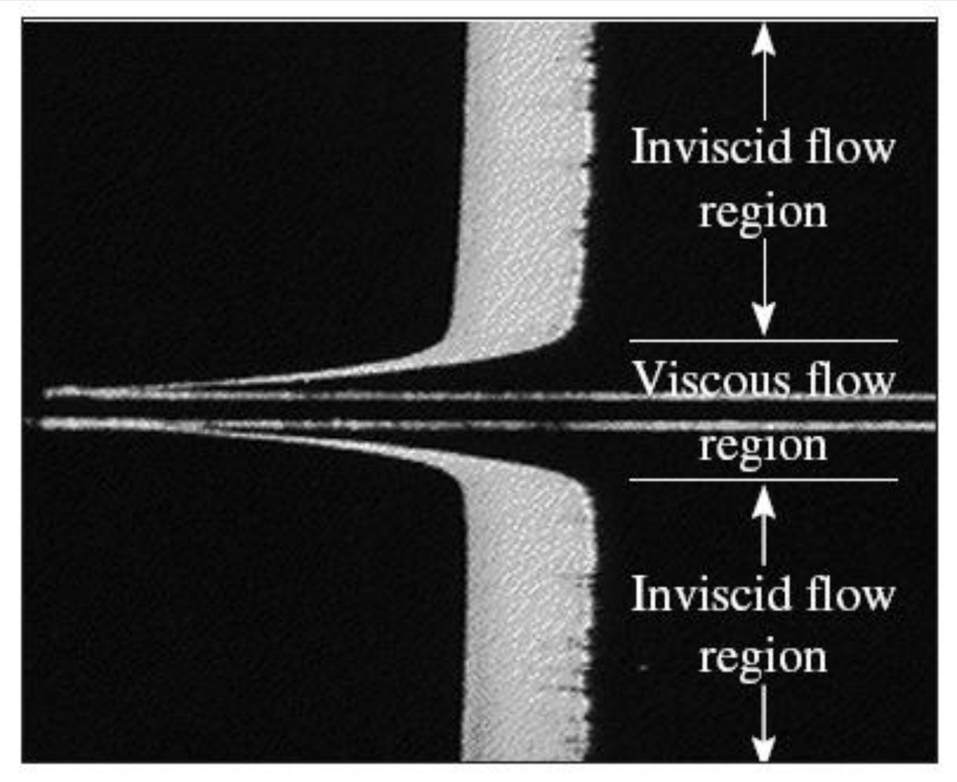
- We turn on the fan on hot summer days to help our body cool more effectively. The higher the fan speed, the better we feel.
- We *stir* our soup and *blow* on a hot slice of pizza to make them cool faster.
- The air on windy winter days feels much colder than it actually is.
- The simplest solution to heating problems in electronics packaging is to use a large enough fan.

CLASSIFICATION OF FLUID FLOWS

Viscous versus Inviscid Regions of Flow

Viscous flows: Flows in which the frictional effects are significant.

Inviscid flow regions: In many flows of practical interest, there are *regions* (typically regions not close to solid surfaces) where viscous forces are negligibly small compared to inertial or pressure forces.

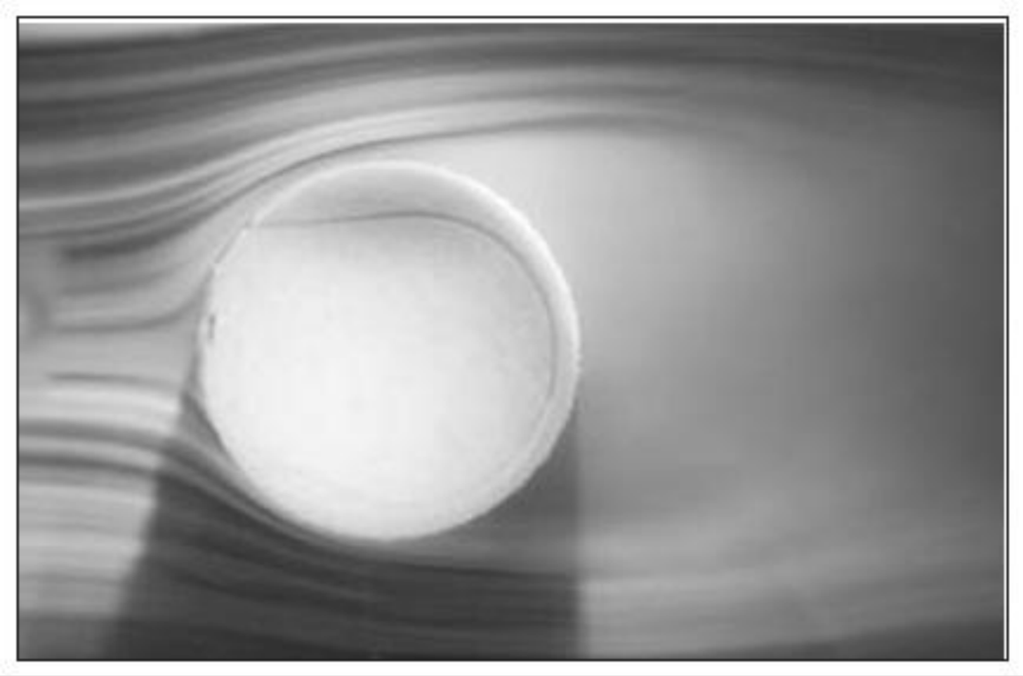


The flow of an originally uniform fluid stream over a flat plate, and the regions of viscous flow (next to the plate on both sides) and inviscid flow (away from the plate).

Internal versus External Flow

External flow: The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe.

Internal flow: The flow in a pipe or duct if the fluid is completely bounded by solid surfaces.



External flow over a tennis ball, and the turbulent wake region behind.

- Water flow in a pipe is internal flow, and airflow over a ball is external flow .
- The flow of liquids in a duct is called *open-channel flow* if the duct is only partially filled with the liquid and there is a free surface.

Compressible versus Incompressible Flow

Incompressible flow: If the density of flowing fluid remains nearly constant throughout (e.g., liquid flow).

Compressible flow: If the density of fluid changes during flow (e.g., high-speed gas flow)

When analyzing rockets, spacecraft, and other systems that involve high-speed gas flows, the flow speed is often expressed by **Mach number**

$$\text{Ma} = \frac{V}{c} = \frac{\text{Speed of flow}}{\text{Speed of sound}}$$

Ma = 1	**Sonic flow**
Ma < 1	**Subsonic flow**
Ma > 1	**Supersonic flow**
Ma >> 1	**Hypersonic flow**

c is the **speed of sound** whose value is 346 m/s in air at room temperature at sea level.

Gas flows can often be approximated as incompressible if the density changes are under about 5 percent, which is usually the case when Ma < 0.3.

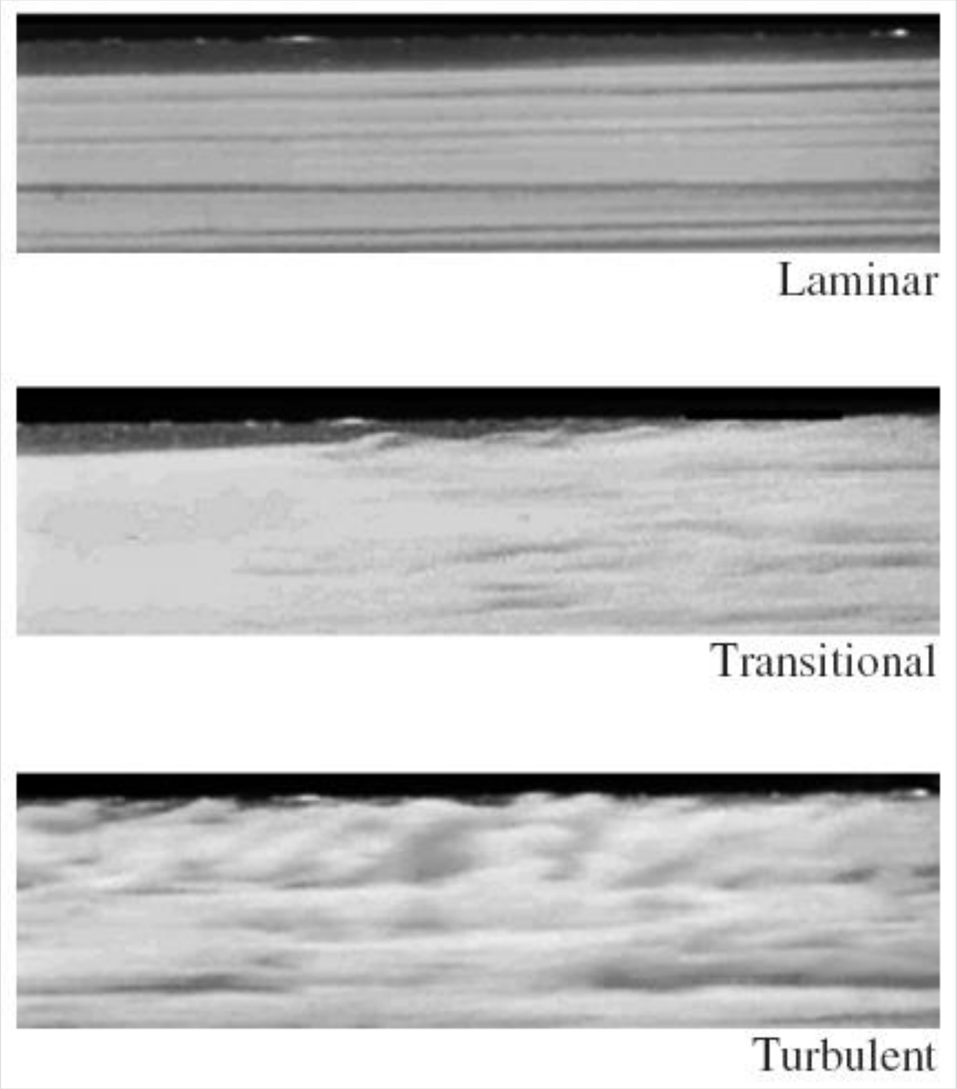
Therefore, the compressibility effects of air can be neglected at speeds under about 100 m/s.

Laminar versus Turbulent Flow

Laminar flow: The highly ordered fluid motion characterized by smooth layers of fluid. The flow of high-viscosity fluids such as oils at low velocities is typically laminar.

Turbulent flow: The highly disordered fluid motion that typically occurs at high velocities and is characterized by velocity fluctuations. The flow of low-viscosity fluids such as air at high velocities is typically turbulent.

Transitional flow: A flow that alternates between being laminar and turbulent.

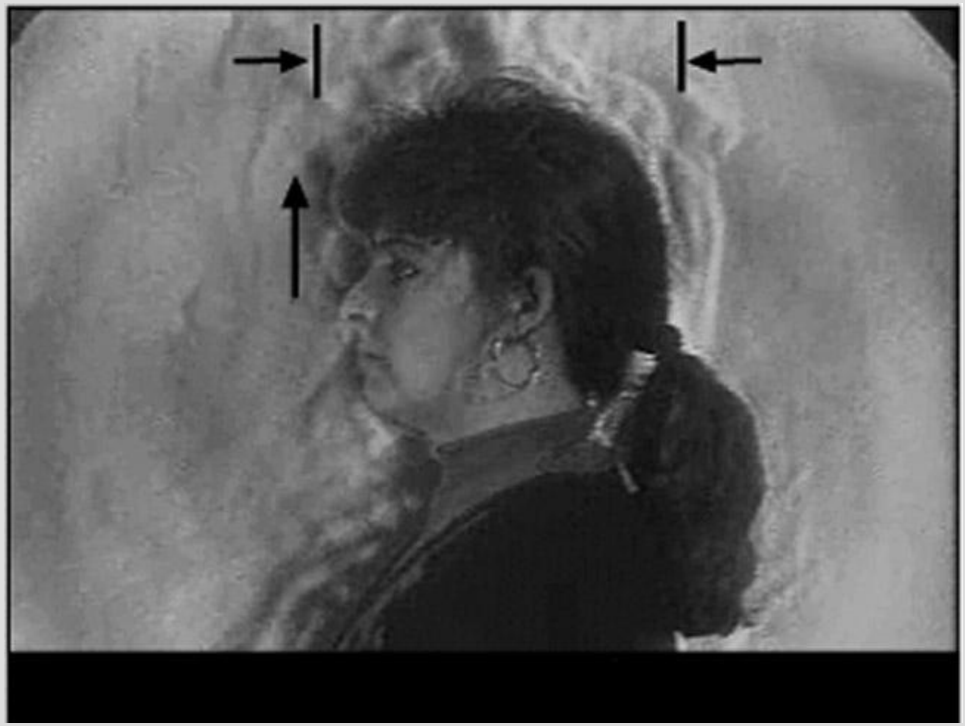


Laminar, transitional, and turbulent flows.

Natural (or Unforced) versus Forced Flow

Forced flow: A fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan.

Natural flow: Fluid motion is due to natural means such as the buoyancy effect, which manifests itself as the rise of warmer (and thus lighter) fluid and the fall of cooler (and thus denser) fluid.



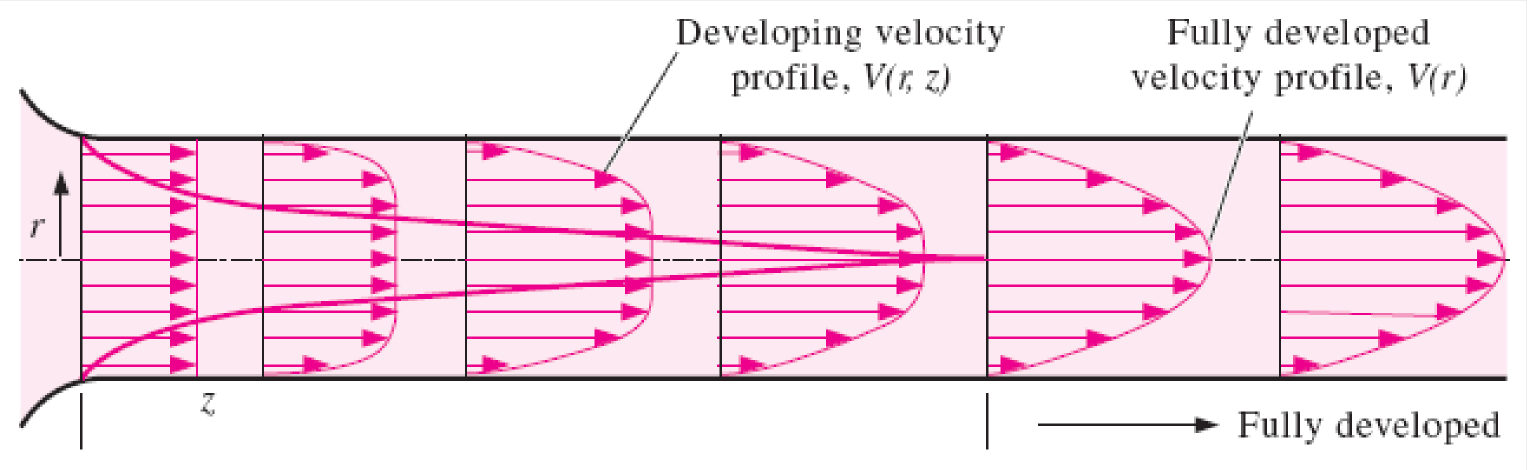
In this schlieren image, the rise of lighter, warmer air adjacent to her body indicates that humans and warm-blooded animals are surrounded by thermal plumes of rising warm air.

Steady versus Unsteady Flow

- The term **steady** implies *no change at a point with time*.
- The opposite of steady is **unsteady**.
- The term **uniform** implies *no change with location* over a specified region.
- The term **periodic** refers to the kind of unsteady flow in which the flow oscillates about a steady mean.
- Many devices such as turbines, compressors, boilers, condensers, and heat exchangers operate for long periods of time under the same conditions, and they are classified as **steady-flow devices**.

One-, Two-, and Three-Dimensional Flows

- A flow field is best characterized by its velocity distribution.
- A flow is said to be one-, two-, or three-dimensional if the flow velocity varies in one, two, or three dimensions, respectively.
- However, the variation of velocity in certain directions can be small relative to the variation in other directions and can be ignored.



The development of the velocity profile in a circular pipe. $V = V(r, z)$ and thus the flow is two-dimensional in the entrance region, and becomes one-dimensional downstream when the velocity profile fully develops and remains unchanged in the flow direction, $V = V(r)$.

VELOCITY BOUNDARY LAYER

Velocity boundary layer: The region of the flow above the plate bounded by δ in which the effects of the viscous shearing forces caused by fluid viscosity are felt.

The *boundary layer thickness*, δ, is typically defined as the distance y from the surface at which $u = 0.99V$.

The hypothetical line of $u = 0.99V$ divides the flow over a plate into two regions:

Boundary layer region: The viscous effects and the velocity changes are significant.

Irrotational flow region: The frictional effects are negligible and the velocity remains essentially constant.

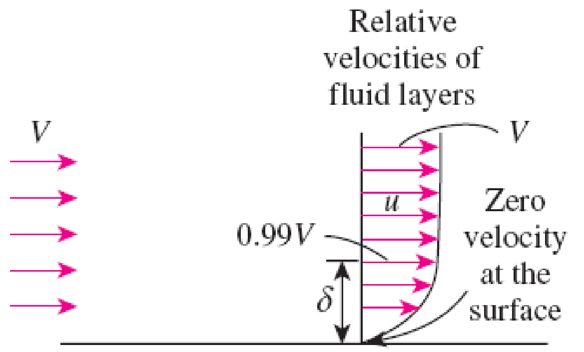


FIGURE 6–15

The development of a boundary layer on a surface is due to the no-slip condition and friction.

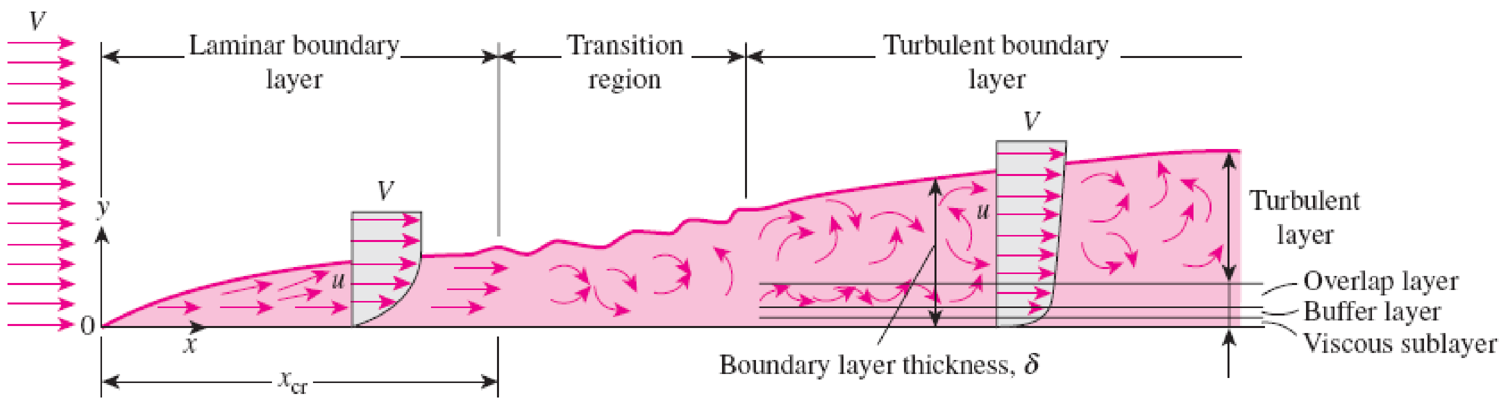


FIGURE 6–14

The development of the boundary layer for flow over a flat plate, and the different flow regimes.

Wall Shear Stress

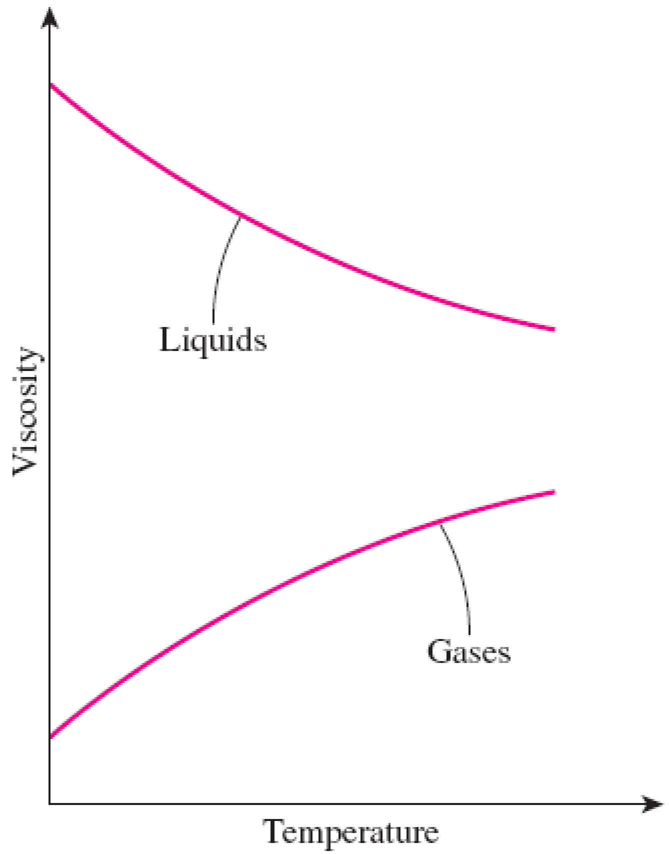


FIGURE 6–16

The viscosity of liquids decreases and the viscosity of gases increases with temperature.

Shear stress: Friction force per unit area.

The shear stress for most fluids is proportional to the *velocity gradient*, and the shear stress at the wall surface is expressed as

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \qquad (\mathrm{N/m^2})$$

μ dynamic viscosity

kg/m·s or N·s/m^2 or Pa·s

1 poise = 0.1 Pa · s

The fluids that obey the linear relationship above are called **Newtonian Fluids**.

Most common fluids such as water, air, gasoline, and oils are Newtonian fluids.

Blood and liquid plastics are examples of non-Newtonian fluids. In this text we consider Newtonian fluids only.

Kinematic viscosity, $\nu = \mu/\rho$

m²/s or stoke

1 stoke = 1 cm²/s = 0.0001 m²/s

The viscosity of a fluid is a measure of its *resistance to deformation*, and it is a strong function of temperature.

Wall shear stress:

$$\tau_w = C_f \frac{\rho V^2}{2} \qquad (\text{N/m}^2)$$

C_f friction coefficient or skin friction coefficient

Friction force over the entire surface:

$$F_f = C_f A_s \frac{\rho V^2}{2} \qquad (\text{N})$$

The friction coefficient is an important parameter in heat transfer studies since it is directly related to the heat transfer coefficient and the power requirements of the pump or fan.

TABLE 6–1

Dynamic viscosities of some fluids at 1 atm and 20°C (unless otherwise stated)

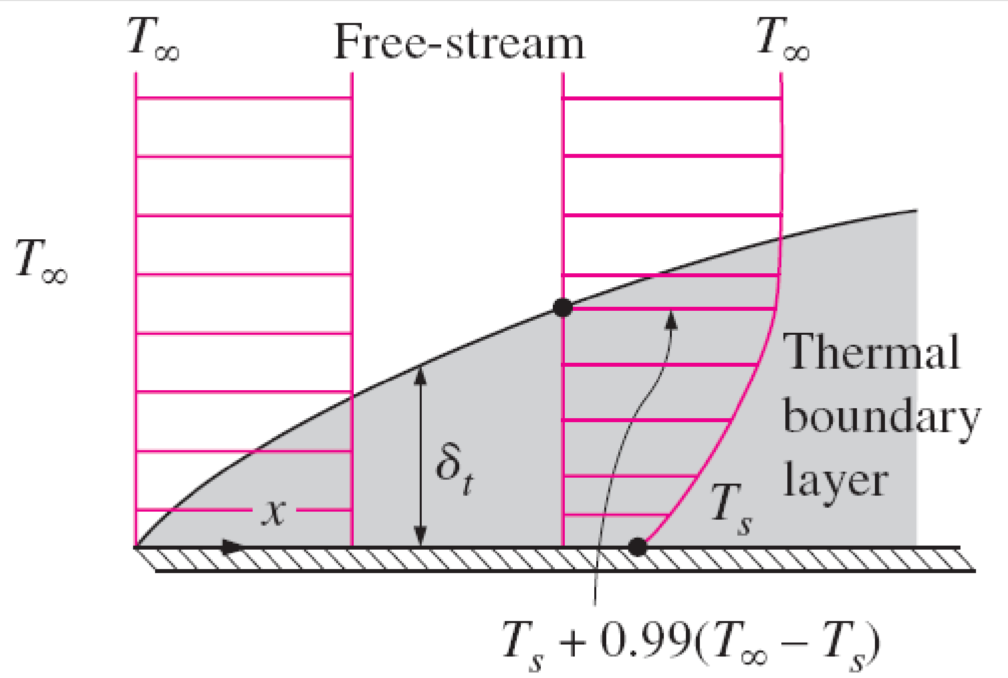
Fluid	Dynamic Viscosity μ, kg/m·s
Glycerin:	
−20°C	134.0
0°C	10.5
20°C	1.52
40°C	0.31
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.00028
100°C (vapor)	0.000012
Blood, 37°C	0.00040
Gasoline	0.00029
Ammonia	0.00015
Air	0.000018
Hydrogen, 0°C	0.0000088

THERMAL BOUNDARY LAYER

A *thermal boundary layer* develops when a fluid at a specified temperature flows over a surface that is at a different temperature.

Thermal boundary layer: The flow region over the surface in which the temperature variation in the direction normal to the surface is significant.

The *thickness* of the thermal boundary layer δ_t at any location along the surface is defined as *the distance from the surface at which the temperature difference $T - T_s$ equals* $0.99(T_\infty - T_s)$.



Thermal boundary layer on a flat plate (the fluid is hotter than the plate surface).

The thickness of the thermal boundary layer increases in the flow direction, since the effects of heat transfer are felt at greater distances from the surface further down stream.

The shape of the temperature profile in the thermal boundary layer dictates the convection heat transfer between a solid surface and the fluid flowing over it.

Prandtl Number

The relative thickness of the velocity and the thermal boundary layers is best described by the *dimensionless* parameter Prandtl number

$$\Pr = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$$

TABLE 6–2

Typical ranges of Prandtl numbers for common fluids

Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.7–1.0
Water	1.7–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000

The Prandtl numbers of gases are about 1, which indicates that both momentum and heat dissipate through the fluid at about the same rate.

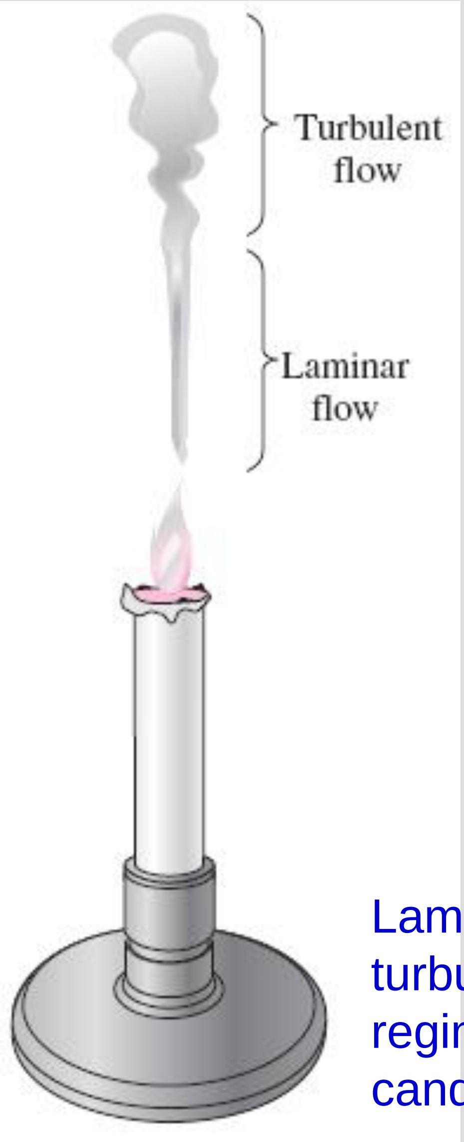
Heat diffuses very quickly in liquid metals ($\Pr \ll 1$) and very slowly in oils ($\Pr \gg 1$) relative to momentum.

Consequently the thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity boundary layer.

Ludwig Prandtl (1875–1953), was a German Physicist famous for his work in aeronautics, born in Freising, Bavaria. His discovery in 1904 of the Boundary Layer which adjoins the surface of a body moving in a fluid led to an understanding of skin friction drag and of the way in which streamlining reduces the drag of airplane wings and other moving bodies. Prandtl's work and decisive advances in boundary layer and wing theories became the basic material of aeronautics. He also made important contributions to the theories of supersonic flow and of turbulence, and contributed much to the development of wind tunnels and other aerodynamic equipment. The dimensionless **Prandtl number** was named after him.

LAMINAR AND TURBULENT FLOWS

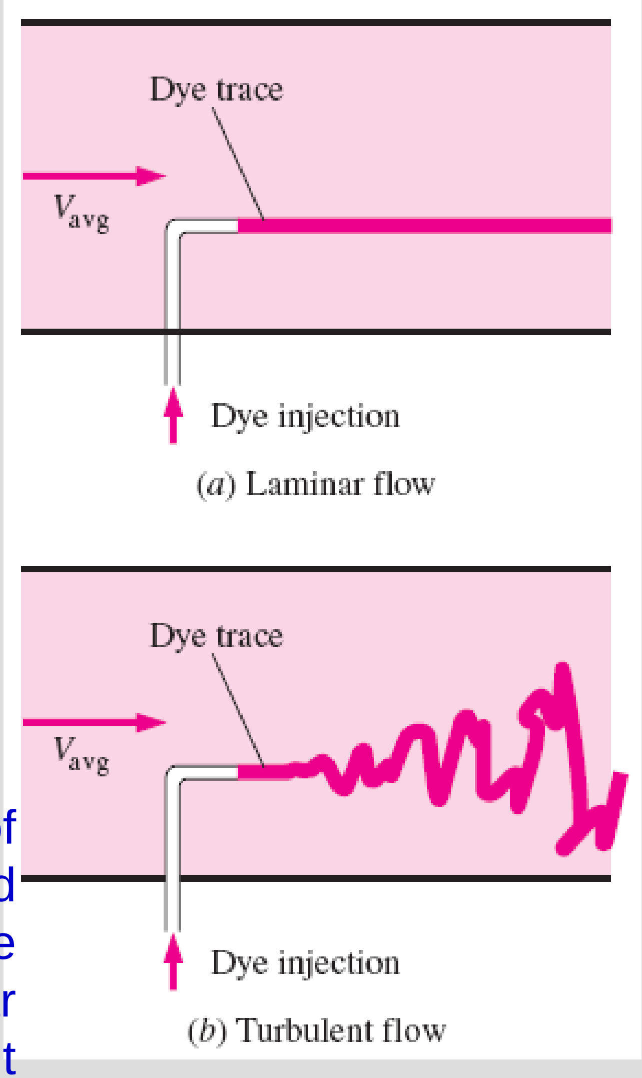
Laminar flow is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages.



Laminar: Smooth streamlines and highly ordered motion.
Turbulent: Velocity fluctuations and highly disordered motion.
Transition: The flow fluctuates between laminar and turbulent flows.

Most flows encountered in practice are turbulent.

Laminar and turbulent flow regimes of candle smoke.



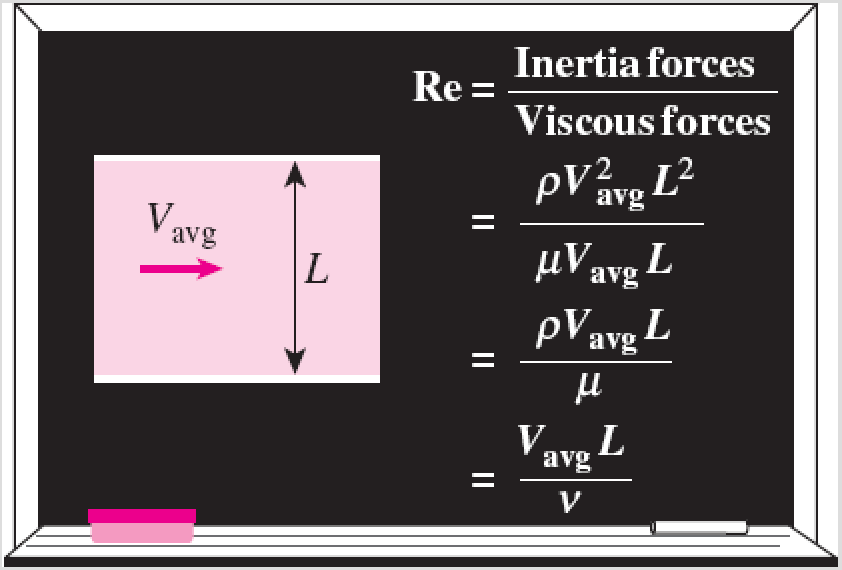
The behavior of colored fluid injected into the flow in laminar and turbulent flows in a pipe.

Reynolds Number

The transition from laminar to turbulent flow depends on the *geometry*, *surface roughness*, *flow velocity*, *surface temperature*, and *type of fluid*.

The flow regime depends mainly on the ratio of *inertia forces* to *viscous forces* (**Reynolds number**).

$$\mathrm{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{avg} D}{\nu} = \frac{\rho V_{avg} D}{\mu}$$



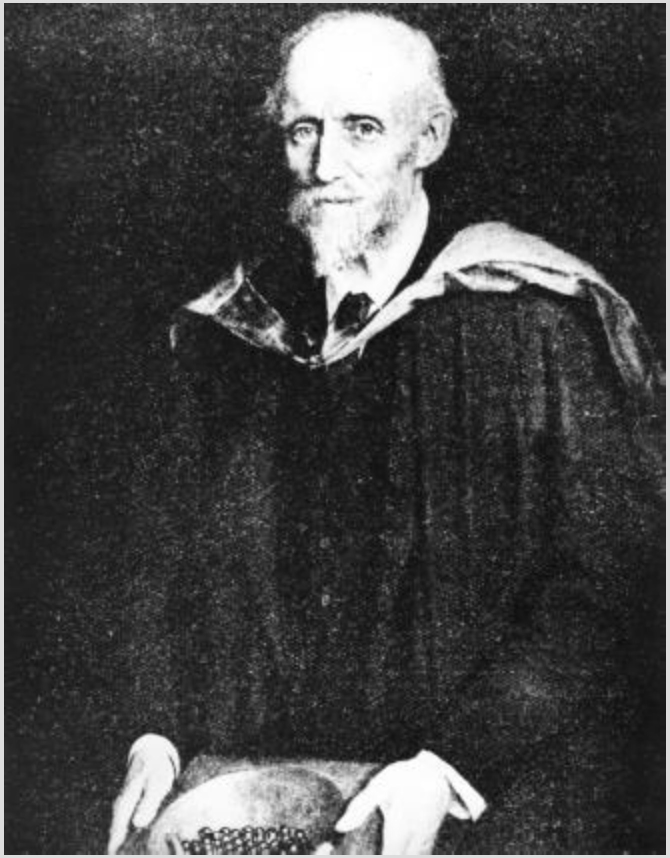
At large Reynolds numbers, the inertial forces, which are proportional to the fluid density and the square of the fluid velocity, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid (**turbulent**).

At small or moderate Reynolds numbers, the viscous forces are large enough to suppress these fluctuations and to keep the fluid "in line" (**laminar**).

Critical Reynolds number, Re_{cr}: The Reynolds number at which the flow becomes turbulent.

The value of the critical Reynolds number is different for different geometries and flow conditions.

The Reynolds number can be viewed as the ratio of inertial forces to viscous forces acting on a fluid element.



Osborne Reynolds (1842–1912), an English engineer and physicist best known for his work in the fields of hydraulics and hydrodynamics, was born in Belfast, Ireland. Reynolds' studies of condensation and the transfer of heat between solids and fluids brought about radical revisions in boiler and condenser design, and his work on turbine pumps laid the foundation for their rapid development. His classical paper on "The Law of Resistance in Parallel Channels" (1883) investigated the transition from smooth, or laminar, to turbulent flow. In 1886 he also formulated "The Theory of Lubrication" and later in 1889, he developed a mathematical framework which became the standard in turbulence work. His other work included the explanation of the radiometer and an early absolute determination of the mechanical equivalent of heat. The dimensionless **Reynolds number**, which provides a criterion for dynamic similarity and for correct modeling in many fluid flow experiments, is named after him.

HEAT AND MOMENTUM TRANSFER IN TURBULENT FLOW

Most flows encountered in engineering practice are turbulent, and thus it is important to understand how turbulence affects wall shear stress and heat transfer.

However, turbulent flow is a complex mechanism dominated by fluctuations, and the theory of turbulent flow is still not fully understood.

Therefore, we must rely on experiments and the empirical or semi-empirical correlations developed for various situations.

Turbulent flow is characterized by disorderly and rapid fluctuations of swirling regions of fluid, called **eddies**, throughout the flow.

These fluctuations provide an additional mechanism for momentum and energy transfer.

The swirling eddies transport mass, momentum, and energy to other regions of flow much more rapidly than molecular diffusion, greatly enhancing mass, momentum, and heat transfer.

Turbulent flow is associated with much higher values of friction, heat transfer, and mass transfer coefficients.

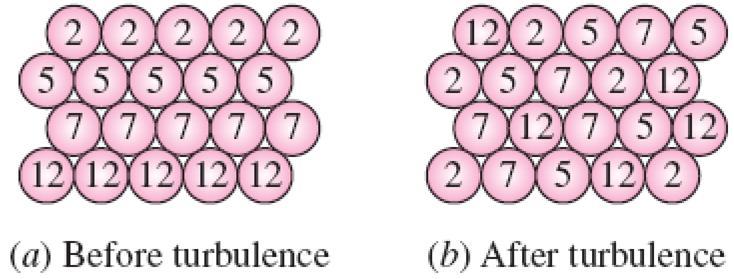


FIGURE 6–23

The intense mixing in turbulent flow brings fluid particles at different temperatures into close contact, and thus enhances heat transfer.

average value $\bar{u}$ and a *fluctuating component* u'. $u = \bar{u} + u'$

$v = \bar{v} + v'$, $P = \bar{P} + P'$, and $T = \bar{T} + T'$

Noting that force in a given direction is equal to the rate of change of momentum in that direction, the horizontal force acting on a fluid element above dA due to the passing of fluid particles through dA is $\delta F = (\rho v' dA)(-u') = -\rho u'v' dA$. Therefore, the shear force per unit area due to the eddy motion of fluid particles $\delta F/dA = -\rho u'v'$ can be viewed as the instantaneous turbulent shear stress. Then the **turbulent shear stress** can be expressed as $\tau_{turb} = -\rho\overline{u'v'}$ where $\overline{u'v'}$ is the time average of the product of the fluctuating velocity components u' and v'. Similarly, considering that $h = c_pT$ represents the energy of the fluid and T' is the eddy temperature relative to the mean value, the rate of thermal energy transport by turbulent eddies is $\dot{q}_{turb} = \rho c_p v'T'$. Note that $\overline{u'v'} \neq 0$ even though $\overline{u'} = 0$ and $\overline{v'} = 0$ (and thus $\overline{u'}\,\overline{v'} = 0$), and experimental results show that $\overline{u'v'}$ is usually a negative quantity. Terms such as $-\rho\overline{u'v'}$ or $-\rho\overline{u'^2}$ are called **Reynolds stresses** or **turbulent stresses**.

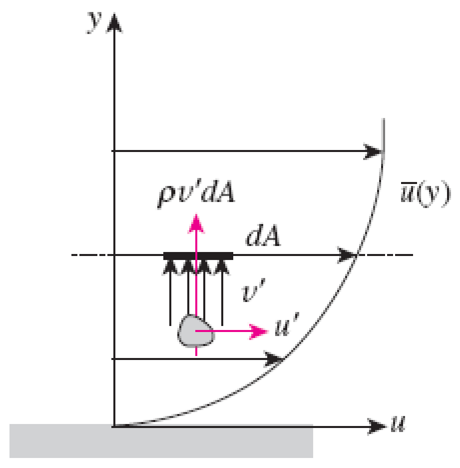


FIGURE 6–25

Fluid particle moving upward through a differential area dA as a result of the velocity fluctuation v'.

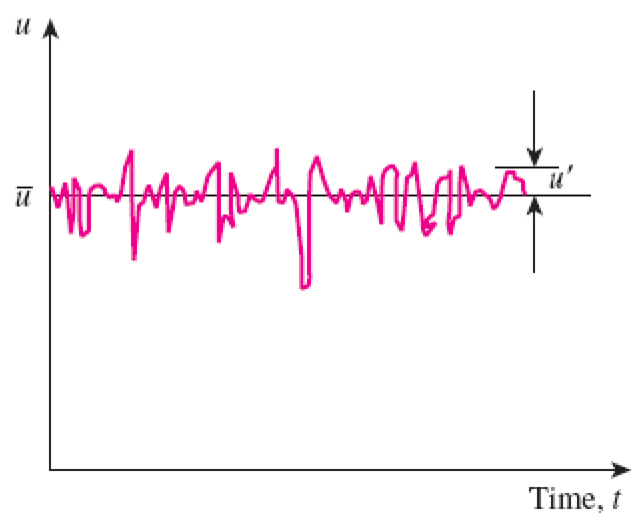


FIGURE 6–24

Fluctuations of the velocity component u with time at a specified location in turbulent flow.

$$\tau_{\text{turb}} = -\rho\,\overline{u'v'} = \mu_t \frac{\partial \overline{u}}{\partial y} \quad \text{and} \quad \dot{q}_{\text{turb}} = \rho c_p\,\overline{vT} = -k_t \frac{\partial T}{\partial y} \tag{6–15}$$

where μ_t is called the **turbulent** (or **eddy**) **viscosity**, which accounts for momentum transport by turbulent eddies, and k_t is called the **turbulent** (or **eddy**) **thermal conductivity**, which accounts for thermal energy transport by turbulent eddies. Then the total shear stress and total heat flux can be expressed conveniently as

$$\tau_{\text{total}} = (\mu + \mu_t)\frac{\partial \overline{u}}{\partial y} = \rho(\nu + \nu_t)\frac{\partial \overline{u}}{\partial y} \tag{6–16}$$

and

$$\dot{q}_{\text{total}} = -(k + k_t)\frac{\partial \overline{T}}{\partial y} = -\rho c_p(\alpha + a_t)\frac{\partial \overline{T}}{\partial y} \tag{6–17}$$

where $\nu_t = \mu_t/\rho$ is the **kinematic eddy viscosity** (or **eddy diffusivity of momentum**) and $\alpha_t = k_t/\rho c_p$ is the **eddy thermal diffusivity** (or **eddy diffusivity of heat**).

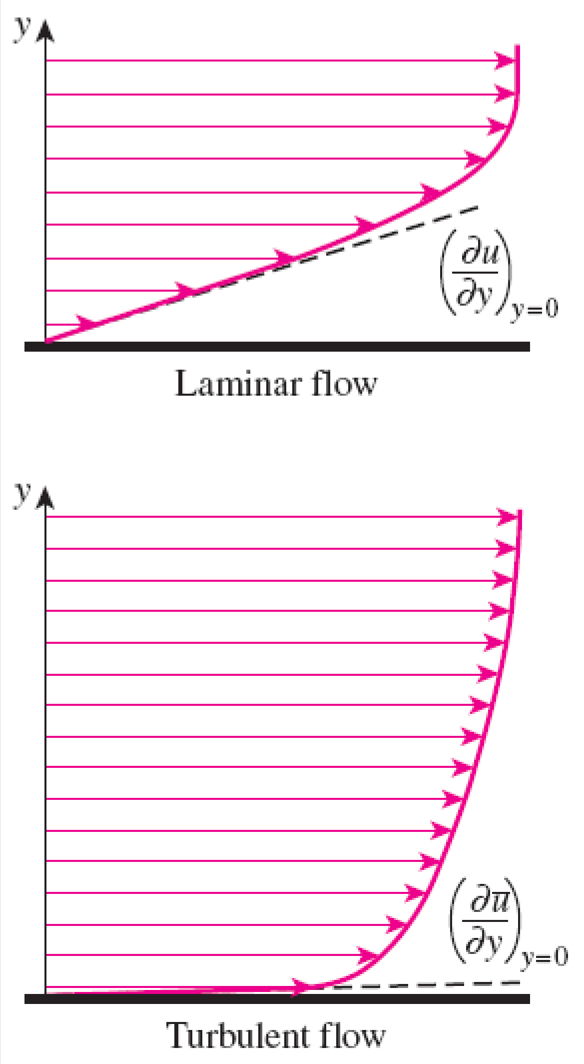


FIGURE 6–26

The velocity gradients at the wall, and thus the wall shear stress, are much larger for turbulent flow than they are for laminar flow, even though the turbulent boundary layer is thicker than the laminar one for the same value of free-stream velocity.

Note that molecular diffusivities ν and α (as well as μ and k) are fluid properties, and their values can be found listed in fluid handbooks. Eddy diffusivities ν_t and α_t (as well as μ_t and k_t), however are *not* fluid properties and their values depend on flow conditions. Eddy diffusivities ν_t and α_t decrease towards the wall, becoming zero at the wall. Their values range from zero at the wall to several thousand times the values of molecular diffusivities in the core region.

DERIVATION OF DIFFERENTIAL CONVECTION EQUATIONS

In this section we derive the governing equations of fluid flow in the boundary layers. To keep the analysis at a manageable level, we assume the flow to be steady and two-dimensional, and the fluid to be Newtonian with constant properties (density, viscosity, thermal conductivity, etc.).

Consider the parallel flow of a fluid over a surface. We take the flow direction along the surface to be x and the direction normal to the surface to be y, and we choose a differential volume element of length dx, height dy, and unit depth in the z-direction (normal to the paper) for analysis (Fig. 6–27). The fluid flows over the surface with a uniform free-stream velocity V, but the velocity within boundary layer is two-dimensional: the x-component of the velocity is u, and the y-component is v. Note that $u = u(x, y)$ and $v = v(x, y)$ in steady two-dimensional flow.

Next we apply three fundamental laws to this fluid element: Conservation of mass, conservation of momentum, and conservation of energy to obtain the continuity, momentum, and energy equations for laminar flow in boundary layers.

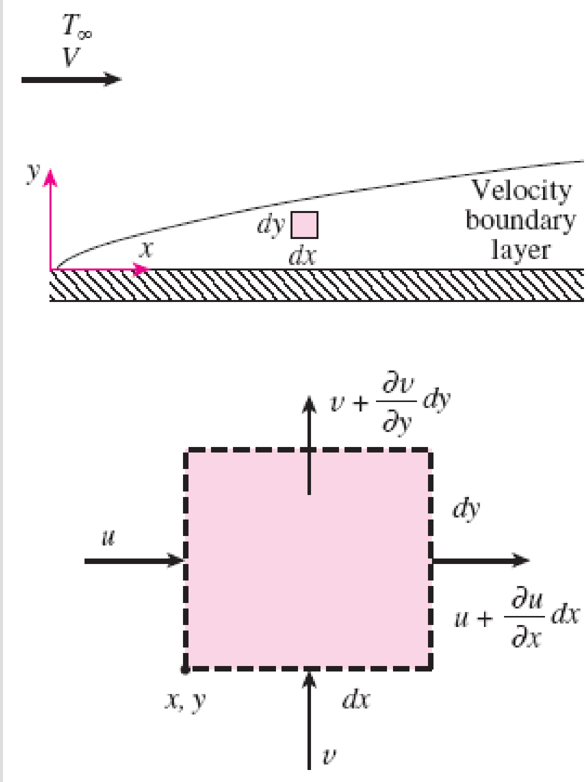


FIGURE 6–27

Differential control volume used in the derivation of mass balance in velocity boundary layer in two-dimensional flow over a surface.

The Continuity Equation

The conservation of mass principle is simply a statement that mass cannot be created or destroyed during a process and all the mass must be accounted for during an analysis. In steady flow, the amount of mass within the control volume remains constant, and thus the conservation of mass can be expressed as

$$\begin{pmatrix} \text{Rate of mass flow} \\ \text{into the control volume} \end{pmatrix} = \begin{pmatrix} \text{Rate of mass flow} \\ \text{out of the control volume} \end{pmatrix} \quad \text{(6–18)}$$

Noting that mass flow rate is equal to the product of density, average velocity, and cross-sectional area normal to flow, the rate at which fluid enters the control volume from the left surface is $\rho u(dy \cdot 1)$. The rate at which the fluid leaves the control volume from the right surface can be expressed as

$$\rho\left(u + \frac{\partial u}{\partial x}dx\right)(dy \cdot 1) \quad \text{(6–19)}$$

Repeating this for the y direction and substituting the results into Eq. 6–18, we obtain

$$\rho u(dy \cdot 1) + \rho v(dx \cdot 1) = \rho\left(u + \frac{\partial u}{\partial x}dx\right)(dy \cdot 1) + \rho\left(v + \frac{\partial v}{\partial y}dy\right)(dx \cdot 1) \quad \text{(6–20)}$$

Simplifying and dividing by $dx \cdot dy \cdot 1$ gives

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{(6–21)}$$

This is the *conservation of mass* relation in differential form, which is also known as the **continuity equation** or **mass balance** for steady two-dimensional flow of a fluid with constant density.

The Momentum Equations

The differential forms of the equations of motion in the velocity boundary layer are obtained by applying Newton's second law of motion to a differential control volume element in the boundary layer. Newton's second law is an expression for momentum balance and can be stated as *the net force acting on the control volume is equal to the mass times the acceleration of the fluid element within the control volume, which is also equal to the net rate of momentum outflow from the control volume.*

We express Newton's second law of motion for the control volume as

$$(\text{Mass})\begin{pmatrix}\text{Acceleration}\\ \text{in a specified direction}\end{pmatrix} = \begin{pmatrix}\text{Net force (body and surface)}\\ \text{acting in that direction}\end{pmatrix} \quad \textbf{(6–22)}$$

or

$$\delta m \cdot a_x = F_{\text{surface},\,x} + F_{\text{body},\,x} \quad \textbf{(6–23)}$$

where the mass of the fluid element within the control volume is

$$\delta m = \rho(dx \cdot dy \cdot 1) \quad \textbf{(6–24)}$$

Noting that flow is steady and two-dimensional and thus $u = u(x, y)$, the total differential of u is

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy \quad \textbf{(6–25)}$$

Then the acceleration of the fluid element in the x direction becomes

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \quad \textbf{(6–26)}$$

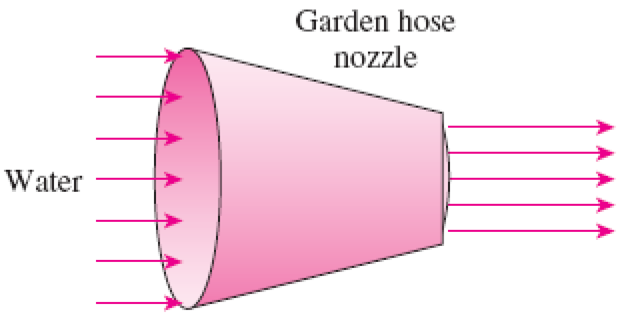


FIGURE 6–28

During steady flow, a fluid may not accelerate in time at a fixed point, but it may accelerate in space.

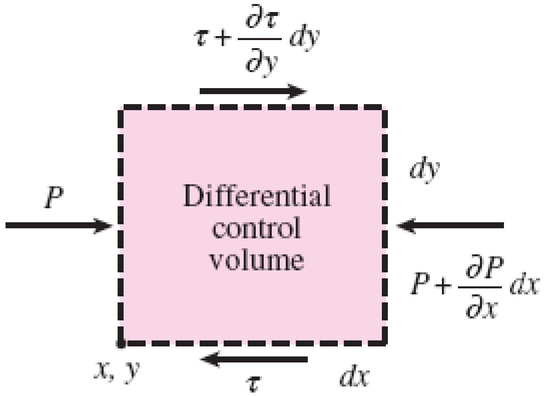


FIGURE 6–29

Differential control volume used in the derivation of x-momentum equation in velocity boundary layer in two-dimensional flow over a surface.

$$F_{\text{surface}, x} = \left(\frac{\partial \tau}{\partial y} dy\right)(dx \cdot 1) - \left(\frac{\partial P}{\partial x} dx\right)(dy \cdot 1) = \left(\frac{\partial \tau}{\partial y} - \frac{\partial P}{\partial x}\right)(dx \cdot dy \cdot 1)$$

$$= \left(\mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}\right)(dx \cdot dy \cdot 1) \qquad \textbf{(6–27)}$$

since $\tau = \mu(\partial u/\partial y)$. Substituting Eqs. 6–24, 6–26, and 6–27 into Eq. 6–23 and dividing by $dx \cdot dy \cdot 1$ gives

$$\rho\left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} \qquad \textbf{(6–28)}$$

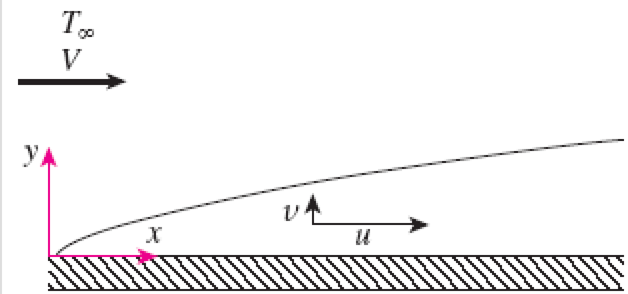
This is the relation for the **momentum balance** in the x-direction, and is known as the **x-momentum equation**. Note that we would obtain the same result if we used momentum flow rates for the left-hand side of this equation instead of mass times acceleration. If there is a body force acting in the x-direction, it can be added to the right side of the equation provided that it is expressed per unit volume of the fluid.

When gravity effects and other body forces are negligible and the boundary layer approximations are valid, applying Newton's second law of motion on the volume element in the y-direction gives the *y-momentum equation* to be

$$\frac{\partial P}{\partial y} = 0 \tag{6–29}$$

That is, *the variation of pressure in the direction normal to the surface is negligible,* and thus $P = P(x)$ and $\partial P/\partial x = dP/dx$. Then it follows that for a given x, the pressure in the boundary layer is equal to the pressure in the free stream, and the pressure determined by a separate analysis of fluid flow in the free stream (which is typically easier because of the absence of viscous effects) can readily be used in the boundary layer analysis.

The velocity components in the free stream region of a flat plate are $u = V =$ constant and $v = 0$. Substituting these into the x-momentum equations (Eq. 6–28) gives $\partial P/\partial x = 0$. Therefore, for flow over a flat plate, the pressure remains constant over the entire plate (both inside and outside the boundary layer).



1) Velocity components:
   $u \gg v$

2) Velocity gradients:
   $\frac{\partial v}{\partial x} \ll 0, \frac{\partial v}{\partial y} \ll 0$

   $\frac{\partial u}{\partial y} \gg \frac{\partial u}{\partial x}$

3) Temperature gradients:
   $\frac{\partial T}{\partial y} \gg \frac{\partial T}{\partial x}$

FIGURE 6–30

Boundary layer approximations.

Conservation of Energy Equation

The energy balance for any system undergoing any process is expressed as $E_{in} - E_{out} = \Delta E_{system}$, which states that the change in the energy content of a system during a process is equal to the difference between the energy input and the energy output. During a *steady-flow process,* the total energy content of a control volume remains constant (and thus $\Delta E_{system} = 0$), and the amount of energy entering a control volume in all forms must be equal to the amount of energy leaving it. Then the rate form of the general energy equation reduces for a steady-flow process to $\dot{E}_{in} - \dot{E}_{out} = 0$.

Noting that energy can be transferred by heat, work, and mass only, the energy balance for a steady-flow control volume can be written explicitly as

$$(\dot{E}_{in} - \dot{E}_{out})_{by\ heat} + (\dot{E}_{in} - \dot{E}_{out})_{by\ work} + (\dot{E}_{in} - \dot{E}_{out})_{by\ mass} = 0 \quad \textbf{(6–30)}$$

The total energy of a flowing fluid stream per unit mass is $e_{stream} = h + \text{ke} + \text{pe}$ where h is the enthalpy (which is the sum of internal energy and flow energy), $\text{pe} = gz$ is the potential energy, and $\text{ke} = V^2/2 = (u^2 + v^2)/2$ is the kinetic energy of the fluid per unit mass. The kinetic and potential energies are usually very small relative to enthalpy, and therefore it is common practice to neglect them (besides, it can be shown that if kinetic energy is included in the following analysis, all the terms due to this inclusion cancel each other). We assume the density ρ, specific heat c_p, viscosity μ, and the thermal conductivity k of the fluid to be constant. Then the energy of the fluid per unit mass can be expressed as $e_{stream} = h = c_pT$.

Energy is a scalar quantity, and thus energy interactions in all directions can be combined in one equation. Noting that mass flow rate of the fluid entering the control volume from the left is $\rho u(dy \cdot 1)$, the rate of energy transfer to the control volume by mass in the x-direction is, from Fig. 6–31,

$$(\dot{E}_{in} - \dot{E}_{out})_{by\ mass,\ x} = (\dot{m}e_{stream})_x - \left[(\dot{m}e_{stream})_x + \frac{\partial(\dot{m}e_{stream})_x}{\partial x}dx\right]$$
$$= -\frac{\partial[\rho u(dy \cdot 1)c_p T]}{\partial x}dx = -\rho c_p\left(u\frac{\partial T}{\partial x} + T\frac{\partial u}{\partial x}\right)dx\,dy \quad \textbf{(6–31)}$$

Repeating this for the y-direction and adding the results, the net rate of energy transfer to the control volume by mass is determined to be

$$(\dot{E}_{in} - \dot{E}_{out})_{by\ mass} = -\rho c_p\left(u\frac{\partial T}{\partial x} + T\frac{\partial u}{\partial x}\right)dx\,dy - \rho c_p\left(v\frac{\partial T}{\partial y} + T\frac{\partial v}{\partial y}\right)dx\,dy$$
$$= -\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)dx\,dy \quad \textbf{(6–32)}$$

since $\partial u/\partial x + \partial v/\partial y = 0$ from the continuity equation.

The net rate of heat conduction to the volume element in the x-direction is

$$(\dot{E}_{in} - \dot{E}_{out})_{by\ heat,\ x} = \dot{Q}_x - \left(\dot{Q}_x + \frac{\partial \dot{Q}_x}{\partial x}dx\right) = -\frac{\partial}{\partial x}\left(-k(dy \cdot 1)\frac{\partial T}{\partial x}\right)dx$$
$$= k\frac{\partial^2 T}{\partial x^2}dx\,dy \quad \textbf{(6–33)}$$

Repeating this for the y-direction and adding the results, the net rate of energy transfer to the control volume by heat conduction becomes

$$(\dot{E}_{in} - \dot{E}_{out})_{by\ heat} = k\frac{\partial^2 T}{\partial x^2}dx\,dy + k\frac{\partial^2 T}{\partial y^2}dx\,dy = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)dx\,dy \quad \textbf{(6–34)}$$

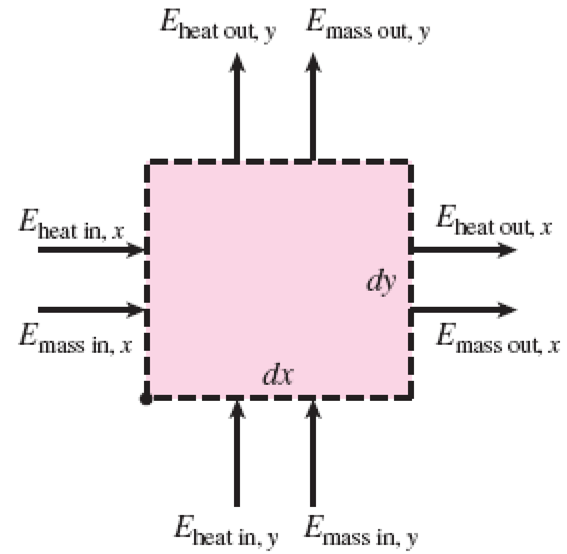


FIGURE 6–31

The energy transfers by heat and mass flow associated with a differential control volume in the thermal boundary layer in steady two-dimensional flow.

Then the energy equation for the steady two-dimensional flow of a fluid with constant properties and negligible shear stresses is obtained by substituting Eqs. 6–32 and 6–34 into 6–30 to be

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{6–35}$$

which states that *the net energy convected by the fluid out of the control volume is equal to the net energy transferred into the control volume by heat conduction.*

When the viscous shear stresses are not negligible, their effect is accounted for by expressing the energy equation as

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\Phi \tag{6–36}$$

where the *viscous dissipation function* Φ is obtained after a lengthy analysis (see an advanced book such as the one by *Schlichting* for details) to be

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \tag{6–37}$$

Viscous dissipation may play a dominant role in high-speed flows, especially when the viscosity of the fluid is high (like the flow of oil in journal bearings). This manifests itself as a significant rise in fluid temperature due to the conversion of the kinetic energy of the fluid to thermal energy. Viscous dissipation is also significant for high-speed flights of aircraft.

For the special case of a stationary fluid, $u = v = 0$, the energy equation reduces, as expected, to the steady two-dimensional heat conduction equation,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{6–38}$$

EXAMPLE 6–1 **Temperature Rise of Oil in a Journal Bearing**

The flow of oil in a journal bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. Such flows are known as Couette flow.

Consider two large isothermal plates separated by 2-mm-thick oil film. The upper plates moves at a constant velocity of 12 m/s, while the lower plate is stationary. Both plates are maintained at 20°C. (*a*) Obtain relations for the velocity and temperature distributions in the oil. (*b*) Determine the maximum temperature in the oil and the heat flux from the oil to each plate (Fig. 6–25).

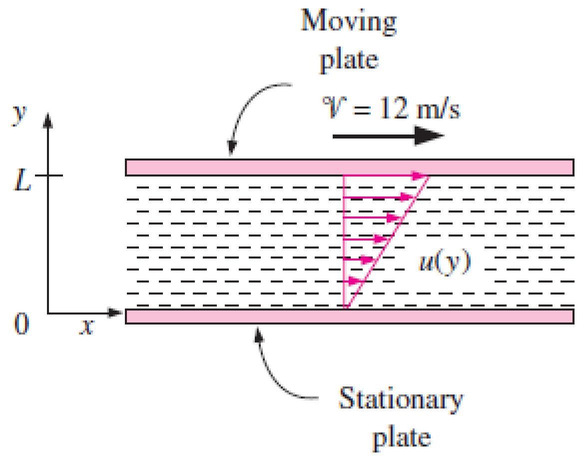


FIGURE 6–25
Schematic for Example 6–1.

SOLUTION Parallel flow of oil between two plates is considered. The velocity and temperature distributions, the maximum temperature, and the total heat transfer rate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible. **4** The plates are large so that there is no variation in the *z* direction.

Properties The properties of oil at 20°C are (Table A-10):

$$k = 0.145 \text{ W/m}\cdot\text{K} \quad \text{and} \quad \mu = 0.800 \text{ kg/m}\cdot\text{s} = 0.800 \text{ N}\cdot\text{s/m}^2$$

Analysis (*a*) We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus $v = 0$. Then the continuity equation (Eq. 6-21) reduces to

$$\textit{Continuity:} \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \rightarrow \frac{\partial u}{\partial x} = 0 \quad \rightarrow u = u(y)$$

Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the *x*-momentum equation (Eq. 6-28) reduces to

$$\textit{x-momentum:}\ \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} \qquad \rightarrow \qquad \frac{d^2u}{dy^2} = 0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y) = C_1 y + C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0) = 0$ and $u(L) = \mathcal{V}$, and applying them gives the velocity distribution to be

$$u(y) = \frac{y}{L}\mathcal{V}$$

Frictional heating due to viscous dissipation in this case is significant because of the high viscosity of oil and the large plate velocity. The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on y only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with dissipation (Eqs. 6-36 and 6-37) reduce to

$$\textit{Energy:} \qquad 0 = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \qquad \rightarrow \qquad k\frac{d^2 T}{dy^2} = -\mu\left(\frac{\mathcal{V}}{L}\right)^2$$

since $\partial u/\partial y = \mathcal{V}/L$. Dividing both sides by k and integrating twice give

$$T(y) = -\frac{\mu}{2k}\left(\frac{y}{L}\mathcal{V}\right)^2 + C_3 y + C_4$$

Applying the boundary conditions $T(0) = T_0$ and $T(L) = T_0$ gives the temperature distribution to be

$$T(y) = T_0 + \frac{\mu\mathcal{V}^2}{2k}\left(\frac{y}{L} - \frac{y^2}{L^2}\right)$$

(b) The temperature gradient is determined by differentiating $T(y)$ with respect to y,

$$\frac{dT}{dy} = \frac{\mu\mathcal{V}^2}{2kL}\left(1 - 2\frac{y}{L}\right)$$

The location of maximum temperature is determined by setting $dT/dy = 0$ and solving for y,

$$\frac{dT}{dy} = \frac{\mu\mathcal{V}^2}{2kL}\left(1 - 2\frac{y}{L}\right) = 0 \qquad \rightarrow \qquad y = \frac{L}{2}$$

Therefore, maximum temperature will occur at mid plane, which is not surprising since both plates are maintained at the same temperature. The maximum temperature is the value of temperature at $y = L/2$,

$$T_{\max} = T\left(\frac{L}{2}\right) = T_0 + \frac{\mu\mathcal{V}^2}{2k}\left(\frac{L/2}{L} - \frac{(L/2)^2}{L^2}\right) = T_0 + \frac{\mu\mathcal{V}^2}{8k}$$

$$= 20 + \frac{(0.8\ \text{N}\cdot\text{s/m}^2)(12\text{m/s})^2}{8(0.145\ \text{W/m}\cdot{}^\circ\text{C})}\left(\frac{1\text{W}}{1\ \text{N}\cdot\text{m/s}}\right) = 119^\circ\text{C}$$

Heat flux at the plates is determined from the definition of heat flux,

$$\dot{q}_0 = -k\frac{dT}{dy}\bigg|_{y=0} = -k\frac{\mu\mathcal{V}^2}{2kL}(1 - 0) = -\frac{\mu\mathcal{V}^2}{2L}$$

$$= -\frac{(0.8\ \text{N}\cdot\text{s/m}^2)(12\ \text{m/s})^2}{2(0.002\ \text{m})}\left(\frac{1W}{1N\cdot m/s}\right) = -28{,}800\ \text{W/m}^2$$

$$\dot{q}_L = -k\frac{dT}{dy}\bigg|_{y=L} = -k\frac{\mu\mathcal{V}^2}{2kL}(1 - 2) = \frac{\mu\mathcal{V}^2}{2L} = -\dot{q}_0 = 28{,}800\ \text{W/m}^2$$

Therefore, heat fluxes at the two plates are equal in magnitude but opposite in sign.

SOLUTIONS OF CONVECTION EQUATIONS FOR A FLAT PLATE

Consider laminar flow of a fluid over a *flat plate,* as shown in Fig. 6–33. Surfaces that are slightly contoured such as turbine blades can also be approximated as flat plates with reasonable accuracy. The x-coordinate is measured along the plate surface from the leading edge of the plate in the direction of the flow, and y is measured from the surface in the normal direction. The fluid approaches the plate in the x-direction with a uniform upstream velocity, which is equivalent to the free stream velocity V.

When viscous dissipation is negligible, the continuity, momentum, and energy equations (Eqs. 6–21, 6–28, and 6–35) reduce for steady, incompressible, laminar flow of a fluid with constant properties over a flat plate to

Continuity:
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \textbf{(6–39)}$$

Momentum:
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \qquad \textbf{(6–40)}$$

Energy:
$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad \textbf{(6–41)}$$

with the boundary conditions (Fig. 6–26)

At $x = 0$: $\quad u(0, y) = V, \quad T(0, y) = T_\infty$

At $y = 0$: $\quad u(x, 0) = 0, \quad v(x, 0) = 0, T(x, 0) = T_w \qquad$ **(6–42)**

As $y \to \infty$: $\quad u(x, \infty) = V, \quad T(x, \infty) = T_\infty$

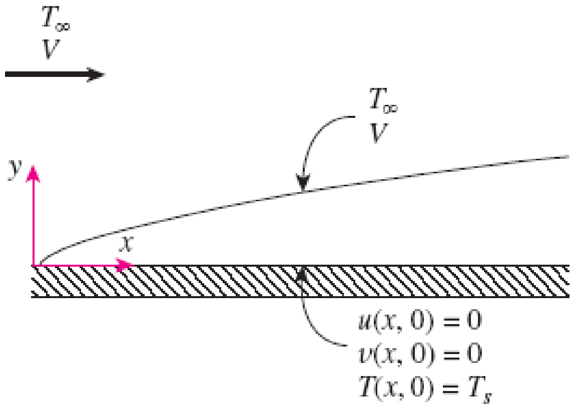


FIGURE 6–33

Boundary conditions for flow over a flat plate.

Noticing that the general shape of the velocity profile remains the same along the plate, Blasius reasoned that the nondimensional velocity profile u/V should remain unchanged when plotted against the nondimensional distance y/δ, where δ is the thickness of the local velocity boundary layer at a given x. That is, although both δ and u at a given y vary with x, the velocity u at a fixed y/δ remains constant. Blasius was also aware from the work of Stokes that δ is proportional to $\sqrt{\nu x/V}$, and thus he defined a *dimensionless similarity variable* as

$$\eta = y\sqrt{\frac{V}{\nu x}} \qquad \textbf{(6–43)}$$

and thus $u/V = \text{function}(\eta)$. He then introduced a *stream function* $\psi(x, y)$ as

$$u = \frac{\partial \psi}{\partial y} \qquad \text{and} \qquad v = -\frac{\partial \psi}{\partial x} \qquad \textbf{(6–44)}$$

so that the continuity equation (Eq. 6–39) is automatically satisfied and thus eliminated (this can be verified easily by direct substitution). Next he defined a function $f(\eta)$ as the dependent variable as

$$f(\eta) = \frac{\psi}{V\sqrt{\nu x/V}} \qquad \textbf{(6–45)}$$

Then the velocity components become

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y} = V\sqrt{\frac{\nu x}{V}}\frac{df}{d\eta}\sqrt{\frac{V}{\nu x}} = V\frac{df}{d\eta} \qquad \textbf{(6–46)}$$

$$v = -\frac{\partial \psi}{\partial x} = -V\sqrt{\frac{\nu x}{V}}\frac{df}{d\eta} - \frac{V}{2}\sqrt{\frac{\nu}{Vx}}f = \frac{1}{2}\sqrt{\frac{V\nu}{x}}\left(\eta\frac{df}{d\eta} - f\right) \qquad \textbf{(6–47)}$$

By differentiating these u and v relations, the derivatives of the velocity components can be shown to be

$$\frac{\partial u}{\partial x} = -\frac{V}{2x}\eta\frac{d^2f}{d\eta^2}, \qquad \frac{\partial u}{\partial y} = V\sqrt{\frac{V}{\nu x}}\frac{d^2f}{d\eta^2}, \qquad \frac{\partial^2 u}{\partial y^2} = \frac{V^2}{\nu x}\frac{d^3f}{d\eta^3} \tag{6–48}$$

Substituting these relations into the momentum equation and simplifying, we obtain

$$2\frac{d^3f}{d\eta^3} + f\frac{d^2f}{d\eta^2} = 0 \tag{6–49}$$

which is a third-order nonlinear differential equation. Therefore, the system of two partial differential equations is transformed into a single ordinary differential equation by the use of a similarity variable. Using the definitions of f and η, the boundary conditions in terms of the similarity variables can be expressed as

$$f(0) = 0, \qquad \left.\frac{df}{d\eta}\right|_{\eta=0} = 0, \qquad \text{and} \qquad \left.\frac{df}{d\eta}\right|_{\eta=\infty} = 1 \tag{6–50}$$

The transformed equation with its associated boundary conditions cannot be solved analytically, and thus an alternative solution method is necessary. The problem was first solved by Blasius in 1908 using a power series expansion approach, and this original solution is known as the *Blasius solution*. The problem is later solved more accurately using different numerical approaches, and results from such a solution are given in Table 6–3. The nondimensional velocity profile can be obtained by plotting u/V against η. The results obtained by this simplified analysis are in excellent agreement with experimental results.

TABLE 6–3

Similarity function f and its derivatives for laminar boundary layer along a flat plate.

η	f	$\frac{df}{d\eta} = \frac{u}{V}$	$\frac{d^2f}{d\eta^2}$
0	0	0	0.332
0.5	0.042	0.166	0.331
1.0	0.166	0.330	0.323
1.5	0.370	0.487	0.303
2.0	0.650	0.630	0.267
2.5	0.996	0.751	0.217
3.0	1.397	0.846	0.161
3.5	1.838	0.913	0.108
4.0	2.306	0.956	0.064
4.5	2.790	0.980	0.034
5.0	3.283	0.992	0.016
5.5	3.781	0.997	0.007
6.0	4.280	0.999	0.002
∞	∞	1	0

Recall that we defined the boundary layer thickness as the distance from the surface for which $u/V = 0.99$. We observe from Table 6–3 that the value of η corresponding to $u/V = 0.99$ is $\eta = 4.91$. Substituting $\eta = 4.91$ and $y = \delta$ into the definition of the similarity variable (Eq. 6–43) gives $4.91 = \delta\sqrt{V/\nu x}$. Then the velocity boundary layer thickness becomes

$$\delta = \frac{4.91}{\sqrt{V/\nu x}} = \frac{4.91x}{\sqrt{\text{Re}_x}} \tag{6–51}$$

since $\text{Re}_x = Vx/\nu$, where x is the distance from the leading edge of the plate. Note that the boundary layer thickness increases with increasing kinematic viscosity ν and with increasing distance from the leading edge x, but it decreases with increasing free-stream velocity V. Therefore, a large free-stream velocity suppresses the boundary layer and causes it to be thinner.

The shear stress on the wall can be determined from its definition and the $\partial u/\partial y$ relation in Eq. 6–48:

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \mu V \sqrt{\frac{V}{\nu x}} \frac{d^2 f}{d\eta^2}\bigg|_{\eta=0} \tag{6–52}$$

Substituting the value of the second derivative of f at $\eta = 0$ from Table 6–3 gives

$$\tau_w = 0.332V\sqrt{\frac{\rho\mu V}{x}} = \frac{0.332\rho V^2}{\sqrt{\text{Re}_x}} \tag{6–53}$$

Then the local friction coefficient becomes

$$C_{f,x} = \frac{\tau_w}{\rho V^2/2} = 0.664\ \text{Re}_x^{-1/2} \tag{6–54}$$

Note that unlike the boundary layer thickness, wall shear stress and the skin friction coefficient decrease along the plate as $x^{-1/2}$.

The Energy Equation

Knowing the velocity profile, we are now ready to solve the energy equation for temperature distribution for the case of constant wall temperature T_s. First we introduce the dimensionless temperature θ as

$$\theta(x, y) = \frac{T(x, y) - T_s}{T_\infty - T_s} \qquad \textbf{(6–55)}$$

Noting that both T_s and T_∞ are constant, substitution into the energy equation Eq. 6–41 gives

$$u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} = \alpha\frac{\partial^2\theta}{\partial y^2} \qquad \textbf{(6–56)}$$

Temperature profiles for flow over an isothermal flat plate are similar, just like the velocity profiles, and thus we expect a similarity solution for temperature to exist. Further, the thickness of the thermal boundary layer is proportional to $\sqrt{\nu x/V}$, just like the thickness of the velocity boundary layer, and thus the similarity variable is also η, and $\theta = \theta(\eta)$. Using the chain rule and substituting the u and v expressions from Eqs. 6–46 and 6–47 into the energy equation gives

$$V\frac{df}{d\eta}\frac{d\theta}{d\eta}\frac{\partial\eta}{\partial x} + \frac{1}{2}\sqrt{\frac{Vy}{x}}\left(\eta\frac{df}{d\eta} f\right)\frac{d\theta}{d\eta}\frac{\partial\eta}{\partial y} = \alpha\frac{d^2\theta}{d\eta^2}\left(\frac{\partial\eta}{\partial y}\right)^2 \qquad \textbf{(6–57)}$$

Simplifying and noting that $\Pr = \nu/\alpha$ gives

$$2\frac{d^2\theta}{d\eta^2} + \Pr f\frac{d\theta}{d\eta} = 0 \qquad \textbf{(6–58)}$$

with the boundary conditions $\theta(0) = 0$ and $\theta(\infty) = 1$. Obtaining an equation for θ as a function of η alone confirms that the temperature profiles are similar, and thus a similarity solution exists. Again a closed-form solution cannot be obtained for this boundary value problem, and it must be solved numerically.

It is interesting to note that for Pr = 1, this equation reduces to Eq. 6–49 when θ is replaced by $df/d\eta$, which is equivalent to u/V (see Eq. 6–46). The boundary conditions for θ and $df/d\eta$ are also identical. Thus we conclude that the velocity and thermal boundary layers coincide, and the nondimensional velocity and temperature profiles (u/V and θ) are identical for steady, incompressible, laminar flow of a fluid with constant properties and Pr = 1 over an isothermal flat plate (Fig. 6–34). The value of the temperature gradient at the surface ($y = 0$ or $\eta = 0$) in this case is, from Table 6–3, $d\theta/d\eta = d^2f/d\eta^2 = 0.332$.

Equation 6–58 is solved for numerous values of Prandtl numbers. For $\Pr > 0.6$, the nondimensional temperature gradient at the surface is found to be proportional to $\Pr^{1/3}$, and is expressed as

$$\left.\frac{d\theta}{d\eta}\right|_{\eta=0} = 0.332 \Pr^{1/3} \tag{6–59}$$

The temperature gradient at the surface is

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = (T_\infty - T_s)\left.\frac{\partial \theta}{\partial y}\right|_{y=0} = (T_\infty - T_s)\left.\frac{d\theta}{d\eta}\right|_{\eta=0}\left.\frac{\partial \eta}{\partial y}\right|_{y=0} \tag{6–60}$$

$$= 0.332 \Pr^{1/3}(T_\infty - T_s)\sqrt{\frac{V}{\nu x}}$$

Then the local convection coefficient and Nusselt number become

$$h_x = \frac{\dot{q}_s}{T_s - T_\infty} = \frac{-k(\partial T/\partial y)|_{y=0}}{T_s - T_\infty} = 0.332 \Pr^{1/3} k\sqrt{\frac{V}{\nu x}} \tag{6–61}$$

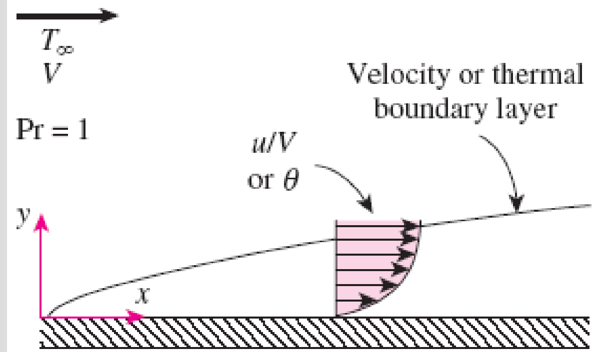


FIGURE 6–34
When Pr = 1, the velocity and thermal boundary layers coincide, and the nondimensional velocity and temperature profiles are identical for steady, incompressible, laminar flow over a flat plate.

and

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\ \mathrm{Pr}^{1/3}\mathrm{Re}_x^{1/2} \qquad \mathrm{Pr} > 0.6 \tag{6–62}$$

The Nu_x values obtained from this relation agree well with measured values.

Solving Eq. 6–58 numerically for the temperature profile for different Prandtl numbers, and using the definition of the thermal boundary layer, it is determined that $\delta/\delta_t \cong \mathrm{Pr}^{1/3}$. Then the thermal boundary layer thickness becomes

$$\delta_t = \frac{\delta}{\mathrm{Pr}^{1/3}} = \frac{4.91x}{\mathrm{Pr}^{1/3}\sqrt{\mathrm{Re}_x}} \tag{6–63}$$

Note that these relations are valid only for laminar flow over an isothermal flat plate. Also, the effect of variable properties can be accounted for by evaluating all such properties at the film temperature defined as $T_f = (T_s + T_\infty)/2$.

The Blasius solution gives important insights, but its value is largely historical because of the limitations it involves. Today both laminar and turbulent flows over surfaces are routinely analyzed using numerical methods.

NONDIMENSIONALIZED CONVECTION EQUATIONS AND SIMILARITY

When viscous dissipation is negligible, the continuity, momentum, and energy equations for steady, laminar flow of a fluid with constant properties are given by Eqs. 6–28, 6–29, and 6–35.

These equations and the boundary conditions can be nondimensionalized by dividing all dependent and independent variables by relevant and meaningful constant quantities: all lengths by a characteristic length L (which is the length for a plate), all velocities by a reference velocity V (which is the free stream velocity for a plate), pressure by ρV^2 (which is twice the free stream dynamic pressure for a plate), and temperature by a suitable temperature difference (which is $T_\infty - T_s$ for a plate). We get

$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}, \quad u^* = \frac{u}{V}, \quad v^* = \frac{v}{V}, \quad P^* = \frac{P}{\rho V^2}, \quad \text{and} \quad T^* = \frac{T - T_s}{T_\infty - T_s}$$

where the asterisks are used to denote nondimensional variables. Introducing these variables into Eqs. 6–28, 6–29, and 6–35 and simplifying give

Continuity:
$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{6–64}$$

Momentum:
$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{dP^*}{dx^*} \tag{6–65}$$

Energy:
$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L \mathrm{Pr}} \frac{\partial^2 T^*}{\partial y^{*2}} \tag{6–66}$$

with the boundary conditions

$$u^*(0, y^*) = 1, \quad u^*(x^*, 0) = 0, \quad u^*(x^*, \infty) = 1, \quad v^*(x^*, 0) = 0, \tag{6–67}$$
$$T^*(0, y^*) = 1, \quad T^*(x^*, 0) = 0, \quad T^*(x^*, \infty) = 1$$

where $Re_L = VL/\nu$ is the dimensionless Reynolds number and $Pr = \nu/\alpha$ is the Prandtl number. For a given type of geometry, the solutions of problems with the same Re and Nu numbers are similar, and thus Re and Nu numbers serve as *similarity parameters.* Two physical phenomena are *similar* if they have the same dimensionless forms of governing differential equations and boundary conditions (Fig. 6–35).

A major advantage of nondimensionalizing is the significant reduction in the number of parameters. The original problem involves 6 parameters (L, V, T_∞, T_s, ν, α), but the nondimensionalized problem involves just 2 parameters (Re_L and Pr). For a given geometry, problems that have the same values for the similarity parameters have identical solutions. For example, determining the convection heat transfer coefficient for flow over a given surface requires numerical solutions or experimental investigations for several fluids, with several sets of velocities, surface lengths, wall temperatures, and free stream temperatures. The same information can be obtained with far fewer investigations by grouping data into the dimensionless Re and Pr numbers. Another advantage of similarity parameters is that they enable us to group the results of a large number of experiments and to report them conveniently in terms of such parameters (Fig. 6–36).

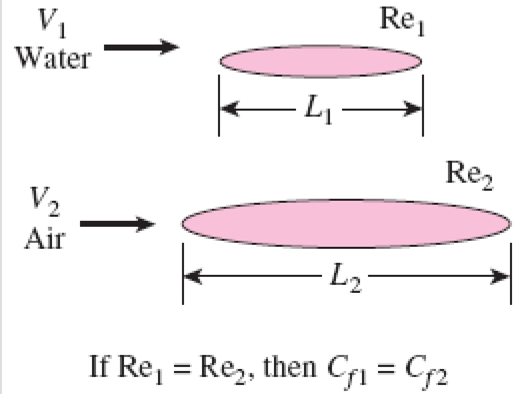


FIGURE 6–35
Two geometrically similar bodies have the same value of friction coefficient at the same Reynolds number.

Parameters before nondimensionalizing
$L, V, T_\infty, T_s, \nu, \alpha$
Parameters after nondimensionalizing:
Re, Pr

FIGURE 6–36
The number of parameters is reduced greatly by nondimensionalizing the convection equations.

FUNCTIONAL FORMS OF FRICTION AND CONVECTION COEFFICIENTS

The three nondimensionalized boundary layer equations (Eqs. 6–64, 6–65, and 6–66) involve three unknown functions u^*, v^*, and T^*, two independent variables x^* and y^*, and two parameters Re_L and Pr. The pressure $P^*(x^*)$ depends on the geometry involved (it is constant for a flat plate), and it has the same value inside and outside the boundary layer at a specified x^*. Therefore, it can be determined separately from the free stream conditions, and dP^*/dx^* in Eq. 6–65 can be treated as a known function of x^*. Note that the boundary conditions do not introduce any new parameters.

For a given geometry, the solution for u^* can be expressed as

$$u^* = f_1(x^*, y^*, \mathrm{Re}_L) \qquad \textbf{(6–68)}$$

Then the shear stress at the surface becomes

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \frac{\mu V}{L} \frac{\partial u^*}{\partial y^*}\bigg|_{y^*=0} = \frac{\mu V}{L} f_2(x^*, \mathrm{Re}_L) \qquad \textbf{(6–69)}$$

Substituting into its definition gives the local friction coefficient,

$$C_{f,x} = \frac{\tau_w}{\rho V^2/2} = \frac{\mu V/L}{\rho V^2/2} f_2(x^*, \mathrm{Re}_L) = \frac{2}{\mathrm{Re}_L} f_2(x^*, \mathrm{Re}_L) = f_3(x^*, \mathrm{Re}_L) \qquad \textbf{(6–70)}$$

Thus we conclude that the friction coefficient for a given geometry can be expressed in terms of the Reynolds number Re and the dimensionless space variable x^* alone (instead of being expressed in terms of x, L, V, ρ, and μ). This is a very significant finding, and shows the value of nondimensionalized equations.

Similarly, the solution of Eq. 6–66 for the dimensionless temperature T^* for a given geometry can be expressed as

$$T^* = g_1(x^*, y^*, \mathrm{Re}_L, \mathrm{Pr}) \tag{6–71}$$

Using the definition of T^*, the convection heat transfer coefficient becomes

$$h_x = \frac{-k(\partial T/\partial y)|_{y=0}}{T_s - T_\infty} = \frac{-k(T_\infty - T_s)}{L(T_s - T_\infty)} \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = \frac{k}{L}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} \tag{6–72}$$

Substituting this into the Nusselt number relation gives [or alternately, we can rearrange the relation above in dimensionless form as $hL/k = (\partial T^*/\partial y^*)|_{y^*=0}$ and define the dimensionless group hL/k as the Nusselt number]

$$\mathrm{Nu}_x = \frac{h_x L}{k} = \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = g_2(x^*, \mathrm{Re}_L, \mathrm{Pr}) \tag{6–73}$$

Note that the Nusselt number is equivalent to the *dimensionless temperature gradient at the surface,* and thus it is properly referred to as the dimensionless heat transfer coefficient (Fig. 6–37). Also, the Nusselt number for a given geometry can be expressed in terms of the Reynolds number Re, the Prandtl number Pr, and the space variable x^*, and such a relation can be used for different fluids flowing at different velocities over similar geometries of different lengths.

The average friction and heat transfer coefficients are determined by integrating $C_{f,x}$ and Nu_x over the surface of the given body with respect to x^* from 0 to 1. Integration removes the dependence on x^*, and the average friction coefficient and Nusselt number can be expressed as

$$C_f = f_4(\mathrm{Re}_L) \quad \text{and} \quad \mathrm{Nu} = g_3(\mathrm{Re}_L, \mathrm{Pr}) \tag{6–74}$$

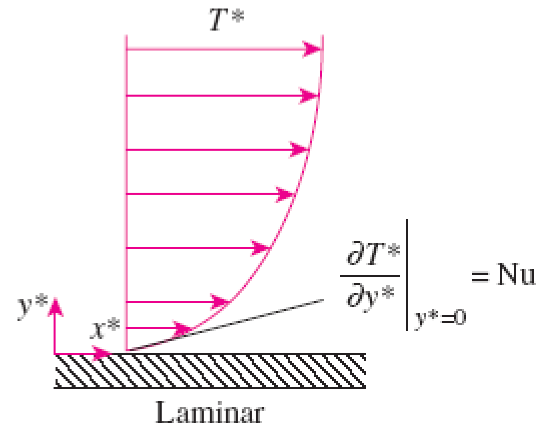


FIGURE 6–37
The Nusselt number is equivalent to the dimensionless temperature gradient at the surface.

These relations are extremely valuable as they state that for a given geometry, the friction coefficient can be expressed as a function of Reynolds number alone, and the Nusselt number as a function of Reynolds and Prandtl numbers alone (Fig. 6–38). Therefore, experimentalists can study a problem with a minimum number of experiments, and report their friction and heat transfer coefficient measurements conveniently in terms of Reynolds and Prandtl numbers. For example, a friction coefficient relation obtained with air for a given surface can also be used for water at the same Reynolds number. But it should be kept in mind that the validity of these relations is limited by the limitations on the boundary layer equations used in the analysis.

The experimental data for heat transfer is often represented with reasonable accuracy by a simple power-law relation of the form

$$\text{Nu} = C\,\text{Re}_L^m\,\text{Pr}^n \quad \text{(6–75)}$$

where m and n are constant exponents (usually between 0 and 1), and the value of the constant C depends on geometry. Sometimes more complex relations are used for better accuracy.

Local Nusselt number:

$$\text{Nu}_x = \text{function}\,(x^*, \text{Re}_L, \text{Pr})$$

Average Nusselt number:

$$\text{Nu} = \text{function}\,(\text{Re}_L, \text{Pr})$$

A common form of Nusselt number:

$$\text{Nu} = C\,\text{Re}_L^m\text{Pr}^n$$

FIGURE 6–38

For a given geometry, the average Nusselt number is a function of Reynolds and Prandtl numbers.

ANALOGIES BETWEEN MOMENTUM AND HEAT TRANSFER

In forced convection analysis, we are primarily interested in the determination of the quantities C_f (to calculate shear stress at the wall) and Nu (to calculate heat transfer rates). Therefore, it is very desirable to have a relation between C_f and Nu so that we can calculate one when the other is available. Such relations are developed on the basis of the similarity between momentum and heat transfers in boundary layers, and are known as *Reynolds analogy* (Fig. 6–21) and *Chilton–Colburn analogy.*

Reconsider the nondimensionalized momentum and energy equations for steady, incompressible, laminar flow of a fluid with constant properties and negligible viscous dissipation (Eqs. 6–65 and 6–66). When Pr = 1 (which is approximately the case for gases) and $\partial P^*/\partial x^* = 0$ (which is the case when, $u = V = $ *constant* in the free stream, as in flow over a flat plate), these equations simplify to

Momentum:
$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{1}{\text{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}} \quad \textbf{(6–76)}$$

Energy:
$$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{1}{\text{Re}_L}\frac{\partial^2 T^*}{\partial y^{*2}} \quad \textbf{(6–77)}$$

which are exactly of the same form for the dimensionless velocity u^* and temperature T^*. The boundary conditions for u^* and T^* are also identical. Therefore, the functions u^* and T^* must be identical, and thus the first derivatives of u^* and T^* at the surface must be equal to each other,

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} \quad \textbf{(6–78)}$$

Then from Eqs. 6–69, 6–70, and 6–73 we have

$$C_{f,x}\frac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x \qquad (\mathrm{Pr} = 1) \tag{6–79}$$

which is known as the **Reynolds analogy** (Fig. 6–39). This is an important analogy since it allows us to determine the heat transfer coefficient for fluids with Pr ≈ 1 from a knowledge of friction coefficient which is easier to measure. Reynolds analogy is also expressed alternately as

$$\frac{C_{f,x}}{2} = \mathrm{St}_x \qquad (\mathrm{Pr} = 1) \tag{6–80}$$

where

$$\mathrm{St} = \frac{h}{\rho c_p V} = \frac{\mathrm{Nu}}{\mathrm{Re}_L \mathrm{Pr}} \tag{6–81}$$

is the **Stanton number** (Fig. 6–40), which is also a dimensionless heat transfer coefficient.

Reynolds analogy is of limited use because of the restrictions Pr = 1 and $\partial P^*/\partial x^* = 0$ on it, and it is desirable to have an analogy that is applicable over a wide range of Pr. This is done by adding a Prandtl number correction.

The friction coefficient and Nusselt number for a flat plate were determined in Section 6–8 to be

$$C_{f,x} = 0.664\,\mathrm{Re}_x^{-1/2} \qquad \text{and} \qquad \mathrm{Nu}_x = 0.332\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2} \tag{6–82}$$

Taking their ratio and rearranging give the desired relation, known as the **modified Reynolds analogy** or **Chilton–Colburn analogy**,

$$C_{f,x}\frac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x\,\mathrm{Pr}^{-1/3} \qquad \text{or} \qquad \frac{C_{f,x}}{2} = \mathrm{St}_x\,\mathrm{Pr}^{2/3} = j_H \tag{6–83}$$

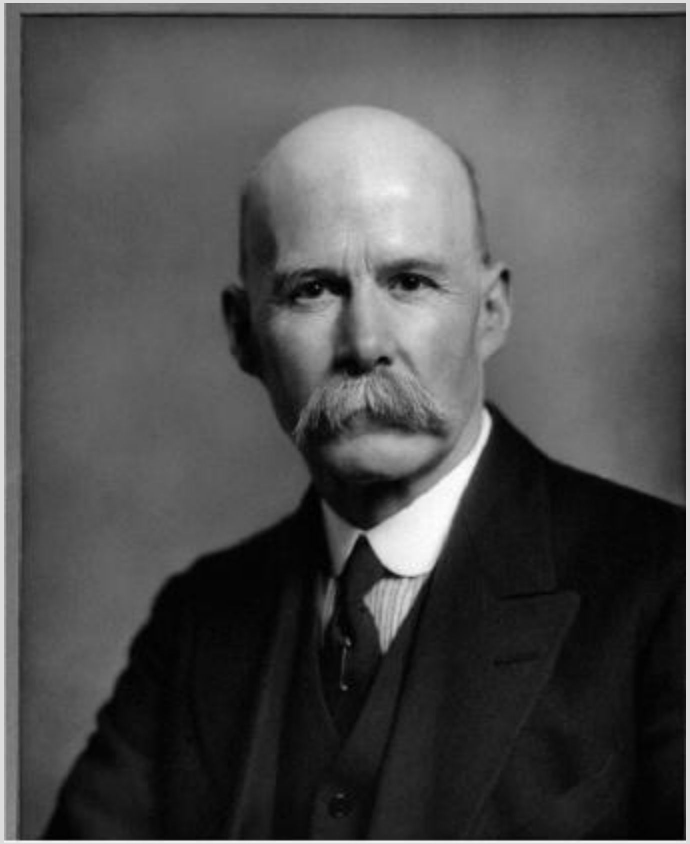
Profiles: $u^* = T^*$

Gradients: $\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$

Analogy: $C_{f,x}\frac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x$

FIGURE 6–39
When Pr = 1 and $\partial P^*/\partial x^* \approx 0$, the nondimensional velocity and temperature profiles become identical, and Nu is related to C_f by Reynolds analogy.

for $0.6 < \Pr < 60$. Here j_H is called the *Colburn j-factor* (Fig. 6–41). Although this relation is developed using relations for laminar flow over a flat plate (for which $\partial P^*/\partial x^* = 0$), experimental studies show that it is also applicable approximately for turbulent flow over a surface, even in the presence of pressure gradients. For laminar flow, however, the analogy is not applicable unless $\partial P^*/\partial x^* \approx 0$. Therefore, it does not apply to laminar flow in a pipe. Analogies between C_f and Nu that are more accurate are also developed, but they are more complex and beyond the scope of this book. The analogies given above can be used for both local and average quantities.



Sir Thomas Edward Stanton (1865–1931), was a British engineer, born at Atherstone in Warwickshire, England. From 1891 to 1896 he worked in Osborne Reynolds' laboratory at Owens College, Manchester, England. Stanton's main field of interest was fluid flow and friction, and the related problem of heat transmission. From 1902 to 1907 he executed a large research program concerning wind forces on structures, such as bridges and roofs. After 1908, the year when the Wright Brothers made their first airplane flights in Europe, Stanton was devoted to problems of airplane and airship design and the dissipation of heat from air-cooled engines. The dimensionless heat transfer coefficient **Stanton number** is named after him.

Allan Philip Colburn (1904–1955), an American engineer, was born in Madison, Wisconsin. His research was on condensation of water vapor from saturated air streams. He brought together for the first time in American engineering work the fundamentals of momentum, heat and mass transfer along with thermodynamic principles to deal with this complex problem. The dimensionless empirical parameter **Colburn j-factor** ($j_H = St_x \mathrm{Pr}^{2/3}$) is named after him.

Some important results from convection equations

The velocity boundary layer thickness

$$\delta = \frac{4.91}{\sqrt{V/\nu x}} = \frac{4.91x}{\sqrt{\mathrm{Re}_x}}$$

The local skin friction coefficient

$$C_{f,x} = \frac{\tau_w}{\rho V^2/2} = 0.664\ \mathrm{Re}_x^{-1/2}$$

Local Nusselt number

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\ \mathrm{Pr}^{1/3}\mathrm{Re}_x^{1/2} \qquad \mathrm{Pr} > 0.6$$

The thermal boundary layer thickness

$$\delta_t = \frac{\delta}{\mathrm{Pr}^{1/3}} = \frac{4.91x}{\mathrm{Pr}^{1/3}\sqrt{\mathrm{Re}_x}}$$

Reynold analogy

$$C_{f,x}\frac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x \qquad (\mathrm{Pr} = 1)$$

$$\frac{C_{f,x}}{2} = \mathrm{St}_x \qquad (\mathrm{Pr} = 1)$$

$$\mathrm{St} = \frac{h}{\rho c_p V} = \frac{\mathrm{Nu}}{\mathrm{Re}_L \mathrm{Pr}}$$

Modified Reynold analogy
or Chilton-Colburn analogy

$$C_{f,x}\frac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x\,\mathrm{Pr}^{-1/3} \qquad \text{or} \qquad \frac{C_{f,x}}{2} = \mathrm{St}_x\,\mathrm{Pr}^{2/3} = j_H$$

EXAMPLE 6–2 Finding Convection Coefficient from Drag Measurement

A 2-m × 3-m flat plate is suspended in a room, and is subjected to air flow parallel to its surfaces along its 3-m-long side. The free stream temperature and velocity of air are 20°C and 7 m/s. The total drag force acting on the plate is measured to be 0.86 N. Determine the average convection heat transfer coefficient for the plate (Fig. 6–33).

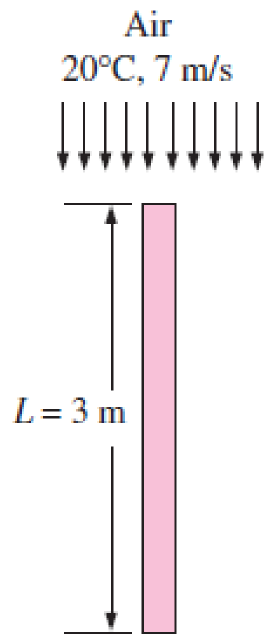


FIGURE 6–33
Schematic for Example 6–2.

SOLUTION A flat plate is subjected to air flow, and the drag force acting on it is measured. The average convection coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 20°C and 1 atm are (Table A-15):

$$\rho = 1.204 \text{ kg/m}^3, \quad C_p = 1.007 \text{ kJ/kg} \cdot \text{K}, \quad \text{Pr} = 0.7309$$

Analysis The flow is along the 3-m side of the plate, and thus the characteristic length is $L = 3$ m. Both sides of the plate are exposed to air flow, and thus the total surface area is

$$A_s = 2WL = 2(2 \text{ m})(3 \text{ m}) = 12 \text{ m}^2$$

For flat plates, the drag force is equivalent to friction force. The average friction coefficient C_f can be determined from Eq. 6-11,

$$F_f = C_f A_s \frac{\rho \mathcal{V}^2}{2}$$

Solving for C_f and substituting,

$$C_f = \frac{F_f}{\rho A_s \mathcal{V}^2/2} = \frac{0.86 \text{ N}}{(1.204 \text{ kg/m}^3)(12 \text{ m}^2)(7 \text{ m/s})^2/2}\left(\frac{1\text{kg} \cdot \text{m/s}^2}{1 \text{ N}}\right) = 0.00243$$

Then the average heat transfer coefficient can be determined from the modified Reynolds analogy (Eq. 6-83) to be

$$h = \frac{C_f}{2}\frac{\rho \mathcal{V} C_p}{\text{Pr}^{2/3}} = \frac{0.00243}{2}\frac{(1.204 \text{ kg/m}^3)(7 \text{ m/s})(1007 \text{ J/kg} \cdot °\text{C})}{0.7309^{2/3}} = 12.7 \text{ W/m}^2 \cdot °\text{C}$$

Discussion This example shows the great utility of momentum-heat transfer analogies in that the convection heat transfer coefficient can be determined from a knowledge of friction coefficient, which is easier to determine.


Merry
Christmas

Summary

- Physical Mechanism of Convection
  - ✓ Nusselt Number
- Classification of Fluid Flows
- Velocity Boundary Layer
  - ✓ Wall shear sttress
- Thermal Boundary Layer
  - ✓ Prandtl Number
- Laminar and Turbulent Flows
  - ✓ Reynolds Number
- Heat and Momentum Transfer in Turbulent Flow
- Derivation of Differential Convection Equations
- Solutions of Convection Equations for a Flat Plate
- Nondimensionalized Convection Equations and Similarity
- Functional Forms of Friction and Convection Coefficients
- Analogies Between Momentum and Heat Transfer